EMTSim verification manual

Every EMTSim model is run on a test circuit and compared with theory. Each section gives the test (as a netlist), a figure, a table of errors, and the cause of every deviation:

kind of deviation	what it is	example
method error	the trapezoidal rule, interpolation, piecewise-linear curves	frequency warping $(\omega\Delta t)^2/12$
model simplification	a property of the chosen model	nominal π vs distributed line
implementation error	the code not doing what the method says	none found

Where possible a result is compared with **two references**: the exact continuous solution (the total error) and the exact solution of the *discretised* equations (agreement to round-off proves the code). Sources are $E(t) = E_{peak}\cos(2\pi ft + angle)$; "pu" is a fraction of the stated reference. Every number and figure is computed by `python docs/manual/make_manual.py` (generated 2026-11-26).

Findings at a glance

error source	size	sections
implementation	every model equals its discretised equations to round-off	all
frequency warping of L and C	$-(\omega\Delta t)^2/12$ in frequency, order 2; the run warns when it exceeds 0.5 %	1.2, 1.3, 11
start-up lag from rest	none with chatter removal (two backward-Euler half steps); $\Delta t/2$ with plain trapezoidal	1.1, 11.2
switching between time points	interpolation + CDA: about 6 % of Δt, order 2	3
network–control interface	none: solved together, order 2 (`coupling=delay` : one step, order 1)	8.1
travelling-wave resampling	$\alpha(1-\alpha)(\omega\Delta t)^2/2$ per traversal when τ is not a whole number of steps	4.3
piecewise-linear arrester	chord error ≤ 0.86 %	6.1
induction machine EMF	previous step's EMF: slip about 1 % high at 100 µs (order 1)	7.2
timers, logic-gated firing	up to one step early; mid-step through a logic block	8.2, 9.3
nominal π line	$(ZY)^2/24$ vs a distributed line	4.1
classical machine with ra	swing mode slightly unstable, as its eigenvalue predicts	7.1
textbook studies	capacitor banks, TRV, line energising, six-pulse bridge: formulas hold, order 2	11

EMTSim and PSCAD compared

PSCAD/EMTDC is the established tool for this kind of study, and EMTSim uses the same family of methods. What differs, as **+** (better for EMTSim), **−** (worse) or **=** (the same):

topic	PSCAD/EMTDC	EMTSim	
solution method	Dommel's nodal method, trapezoidal rule, sparse solver	the same	=
switching between time points	interpolation to the instant	interpolation to the instant (section 3)	=
numerical chatter	half-step interpolation after switchings	the same method (`chatter=pscad`), or CDA, detection, root matching (section 3)	+ choice
control–network interface	controls run after the network: a one-step delay, a first-order phase lag ωΔt	solved together each step: no delay, order 2 (section 8.1); the delay as an option	+
start of a run	sources ramped up; machines started as sources, then released	power-flow steady start of machines and controls (`init=steady`); a run from rest starts without the half-step lag	+
snapshots (save and restart a run)	yes	no	−
time-step guidance	none built in	warns when Δt warps a resonance by more than 0.5 %, with the Δt to use	+
line and cable models	Bergeron; frequency-dependent in modal and phase domain (universal line model); cable constants	Bergeron; frequency-dependent (J. Marti, modal, transposed lines); no cables	−
transformers	classical and core-topology (UMEC) models, many windings	two-winding 1- and 3-phase, vector groups, two-slope saturation	−
machines	detailed synchronous (multi-mass), induction, permanent-magnet and more	classical and model 2.2 (GENROU form), induction	−
power electronics and HVDC	valves and a large converter and HVDC library	valves, IGBTs, average and switching converters, inverter controls	−
control blocks	the CSMF library	about 130 blocks in the CSMF style	=
frequency scans	yes	yes (`.scan`)	=
parallel and multiple runs	yes	one run at a time	−
verification	decades of industrial use and benchmarks	this manual: every model against analytical solutions	+ / −
circuit files	project files of the program	plain-text netlists with the drawing inside: readable, comparable, version-controlled	+
user models	Fortran/C components	one C++ file per component (section 10)	=
cost	commercial licence	free up to 10 nodes; $2.99 and $9.99 a month	+

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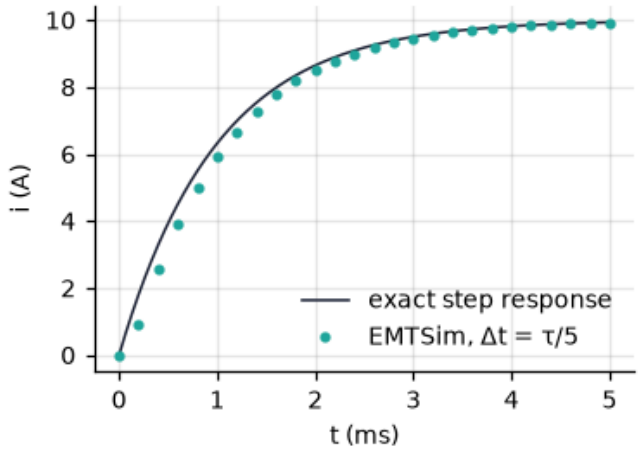
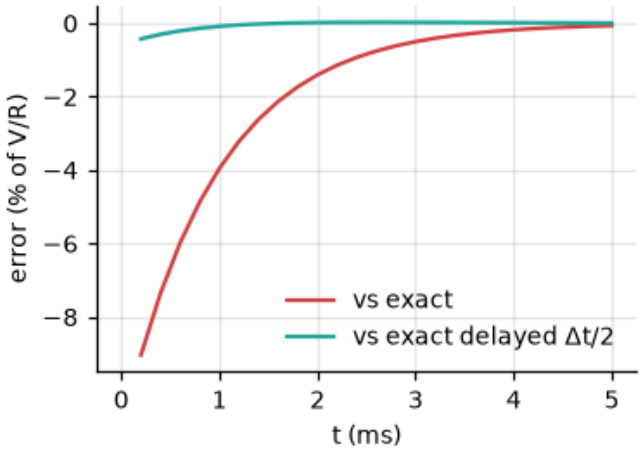
1. Resistor, inductor and capacitor (`R`, `L`, `C`)

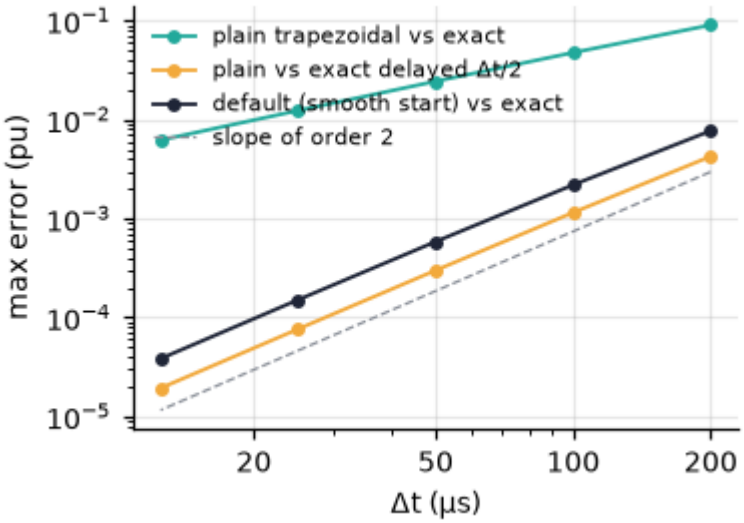
R is exact (a stamp 1/R, no history). L and C use the trapezoidal rule; this chapter measures what it does: the start-up delay, frequency warping, no damping, and the steady-state reactance error.

1.1 Inductor: step response and the start-up delay

Model. `L` becomes the conductance Δt/2L with a history source: $i_k = i_{k-1} + (\Delta t/2L)(v_{k-1} + v_k)$, with zero history at t = 0. **Test:** 10 V DC onto R = 1 Ω, L = 1 mH (τ = 1 ms), Δt = τ/5 … τ/80, plain trapezoidal and default options. **Theory:** $i = (V/R)(1 - e^{-t/\tau})$.







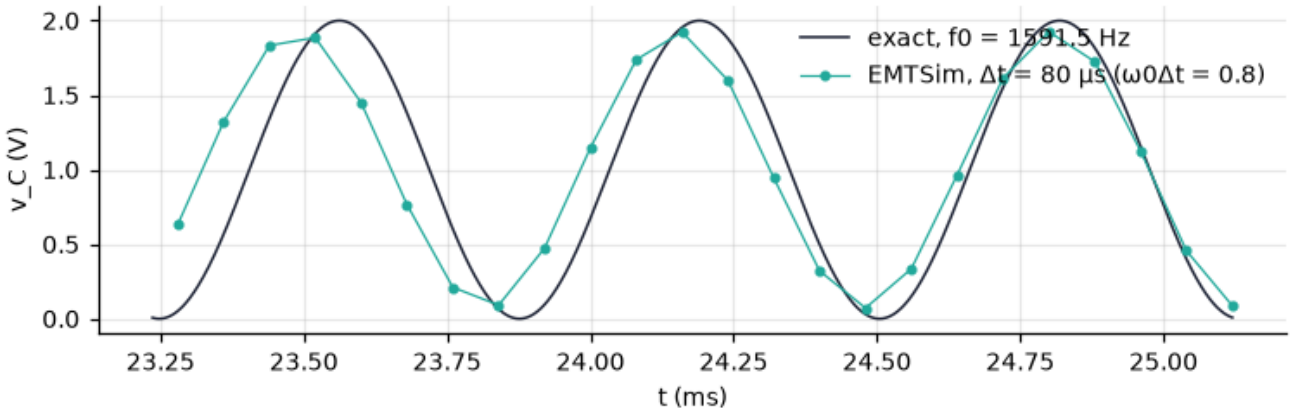
Δt	Δt/τ	plain vs exact (pu)	order	plain vs delayed Δt/2 (pu)	order	plain vs trapezoidal recursion (pu)
200 µs	0.2	0.0904	–	0.00425	–	5.33×10^{-16}
100 µs	0.1	0.0475	0.93	0.00115	1.89	5.33×10^{-16}
50 µs	0.05	0.0244	0.96	3.00×10^{-4}	1.94	1.78×10^{-15}
25 µs	0.025	0.0123	0.98	7.65×10^{-5}	1.97	1.60×10^{-15}
12.5 µs	0.0125	0.00621	0.99	1.93×10^{-5}	1.99	8.35×10^{-15}

- **Code = method:** the plain run equals the trapezoidal recursion to 8.35×10^{-15}.
- **Plain trapezoidal from rest:** order 0.99 against the exact answer, because zero history makes the source look like a ramp over the first step, a delay of Δt/2. Against the delayed response the order is 1.99.

- **Default options:** the run starts with two backward-Euler half steps, so there is no delay: order 1.98 against the exact answer itself.

1.2 Inductor and capacitor together: frequency warping, no numerical damping

Test: 1 V DC onto L = 1 mH and C = 10 µF in series (f_0 = 1591.55 Hz), 40 cycles, five time steps. **Theory:** the trapezoidal rule maps $\pm j\omega_0$ to $e^{\pm j\theta}$, $\theta = 2\,\mathrm{atan}(\omega_0\Delta t/2)$: the amplitude is kept exactly and the frequency is $f_{trap} = \theta/(2\pi\Delta t)$, an error of $-(\omega_0\Delta t)^2/12$. The frequency is fitted from $x_{k+1} + x_{k-1} = 2\cos\theta \cdot x_k$ (no interpolation).

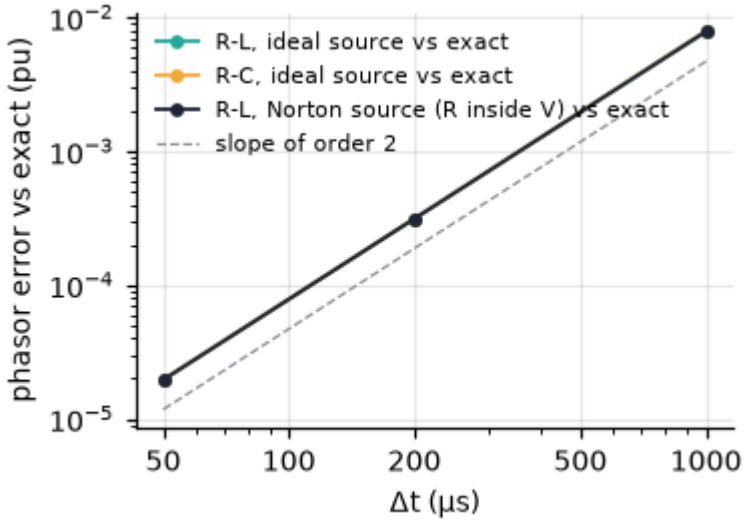


Δt	ω0Δt	f trapezoidal (Hz)	f measured (Hz)	(f_trap − f0)/f0 (%)	measured − theory (pu of f0)	energy drift (pu)
10 µs	0.10	1590.2251	1590.2251	-0.083	-9.00×10^{-15}	1.34×10^{-13}
20 µs	0.20	1586.2759	1586.2759	-0.331	3.14×10^{-15}	5.82×10^{-14}
40 µs	0.40	1570.8240	1570.8240	-1.302	-4.29×10^{-16}	4.07×10^{-14}
80 µs	0.80	1513.9868	1513.9868	-4.873	0	5.84×10^{-14}
160 µs	1.60	1342.3545	1342.3545	-15.657	-1.43×10^{-16}	5.83×10^{-15}

- **Frequency** equals the trapezoidal prediction to 9.00×10^{-15} of f_0: the whole error is the warping (−0.08 % at $\omega_0\Delta t$ = 0.1, −16 % at 1.6).
- **Energy** is conserved to 1.34×10^{-13}: no numerical damping (which is why the rule can chatter, section 3).
- **Guidance:** $\omega_0\Delta t \le 0.18$ keeps the phase error under 1° per cycle ($(\omega_0\Delta t)^2/12$ of 360°; 1.2° at 0.2). The run warns when a resonance is warped by more than 0.5 %.

1.3 Sinusoidal steady state: the trapezoidal reactance

Theory: in steady state s = jω becomes $j(2/\Delta t)\tan(\omega\Delta t/2)$: L and C act at $\omega_{trap} > \omega$, a reactance error of + $(\omega\Delta t)^2/12$ (0.82 % at 50 Hz and Δt = 1 ms, 0.002 % at 50 µs). **Test:** 100 V, 50 Hz with 10 Ω and 0.1 H or 100 µF (and a Norton source), 0.4 s; the current phasor from a DFT of the last cycle.



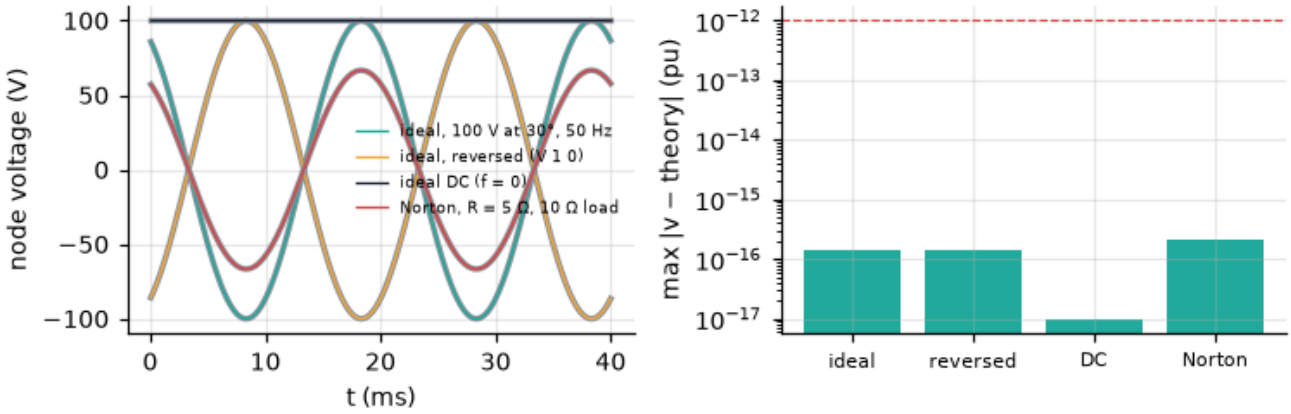
Δt	circuit	|I| exact (A)	|I| at ω_trap (A)	|I| simulated (A)	angle exact (°)	angle simulated (°)	vs exact (pu)	vs ω_trap phasor (pu)
1000 µs	R-L, ideal source	3.03	3.01	3.01	-72.3	-72.5	0.00786	1.90×10^{-15}
1000 µs	R-C, ideal source	3	3.02	3.02	72.6	72.4	0.00792	7.58×10^{-16}
1000 µs	R-L, Norton source (R inside V)	3.03	3.01	3.01	-72.3	-72.5	0.00786	1.90×10^{-15}
200 µs	R-L, ideal source	3.03	3.03	3.03	-72.3	-72.3	3.14×10^{-4}	1.90×10^{-13}
200 µs	R-C, ideal source	3	3	3	72.6	72.6	3.14×10^{-4}	8.90×10^{-14}
200 µs	R-L, Norton source (R inside V)	3.03	3.03	3.03	-72.3	-72.3	3.14×10^{-4}	1.90×10^{-13}
50 µs	R-L, ideal source	3.03	3.03	3.03	-72.3	-72.3	1.96×10^{-5}	1.92×10^{-13}
50 µs	R-C, ideal source	3	3	3	72.6	72.6	1.96×10^{-5}	9.21×10^{-14}
50 µs	R-L, Norton source (R inside V)	3.03	3.03	3.03	-72.3	-72.3	1.96×10^{-5}	1.92×10^{-13}

- **Code = method:** the phasor equals the one at ω_{trap} to 1.92×10^{-13}.
- **Against the exact phasor** the error is the reactance warping alone, order 2.

2. Voltage sources (`V`, `V3`)

2.1 Single-phase source `V`

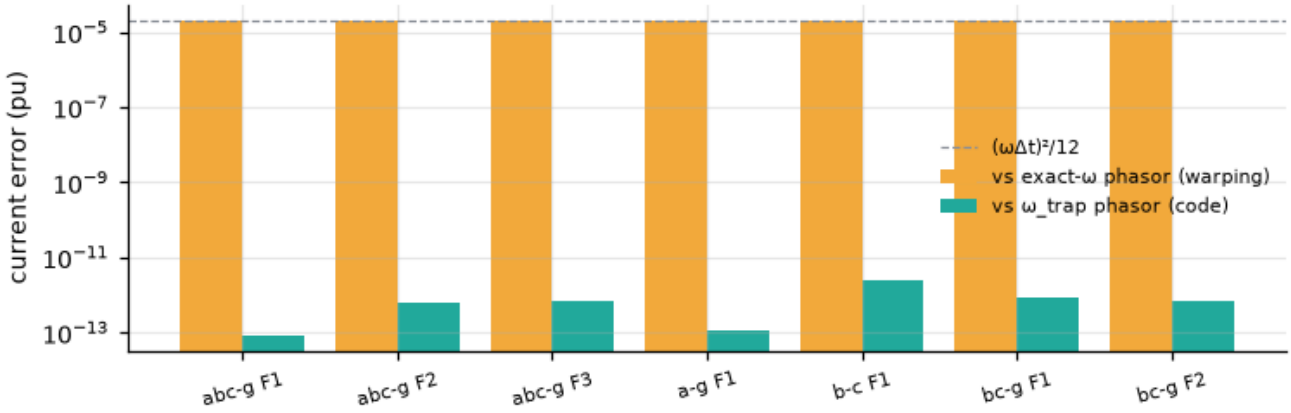
Model: $E(t) = E_{peak}\cos(2\pi ft + \text{angle})$. Without R the source is ideal and fixes a known node (one terminal on ground); with R it is a Norton equivalent; with R and L, an ideal source behind an R–L branch. **Test:** each source into 10 Ω, two cycles, Δt = 50 µs (grey: theory; dashed: the 10^{-12} limit).



- **Exact to round-off:** an algebraic source has no discretisation error; angle and polarity are as stated.

2.2 Three-phase source behind a sequence impedance (`V3 … R1 L1 R0 L0`)

Model: three ideal EMFs (0°, −120°, +120°) behind a coupled R–L matrix from sequence data, $Z_s = (Z_0 + 2Z_1)/3$, $Z_m = (Z_0 - Z_1)/3$, discretised as a whole. **Test:** E = 1000 V, Z_1 = 0.5 Ω + 10 mH, Z_0 = 1.5 Ω + 30 mH, four terminal faults through 0.05 Ω, Δt = 50 µs, 0.5 s; steady-state currents from the last cycle. **Theory:** $(Y_{abc} + Y_f)V = Y_{abc}E$ at ω and at ω_{trap}; textbook: a–g $3E/(2Z_1 + Z_0 + 3R_f)$ = 188.32 A, b–c $\sqrt{3}E/|2Z_1 + R_f|$ = 271.89 A.



fault	branch	|I| exact (A)	|I| simulated (A)	angle exact (°)	angle simulated (°)	vs exact (pu)	vs ω_trap phasor (pu)
three-phase (abc-g)	F1	314	314	-80.1	-80.1	2.03×10^{-5}	8.42×10^{-14}
three-phase (abc-g)	F2	314	314	160	160	2.03×10^{-5}	6.30×10^{-13}
three-phase (abc-g)	F3	314	314	39.9	39.9	2.03×10^{-5}	7.14×10^{-13}
single line to ground (a-g)	F1	188	188	-80.4	-80.4	2.03×10^{-5}	1.10×10^{-13}
line to line (b-c)	F1	272	272	-171	-171	2.03×10^{-5}	2.36×10^{-12}
double line to ground (bc-g)	F1	279	279	176	176	2.02×10^{-5}	8.16×10^{-13}

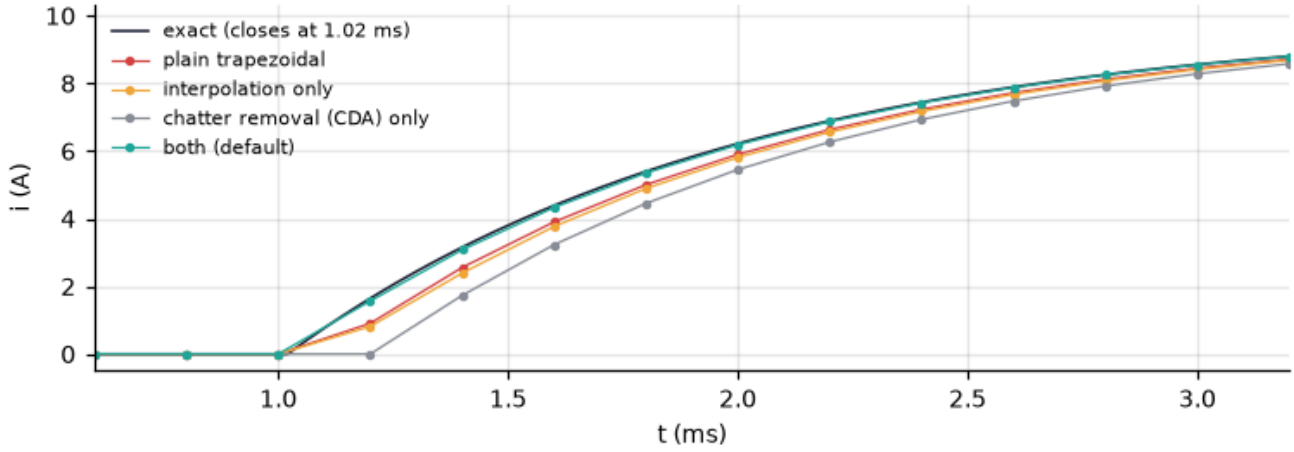
fault	branch	|I| exact (A)	|I| simulated (A)	angle exact (°)	angle simulated (°)	vs exact (pu)	vs ω_trap phasor (pu)
double line to ground (bc-g)	F2	280	280	23.8	23.8	2.03×10^{-5}	6.92×10^{-13}

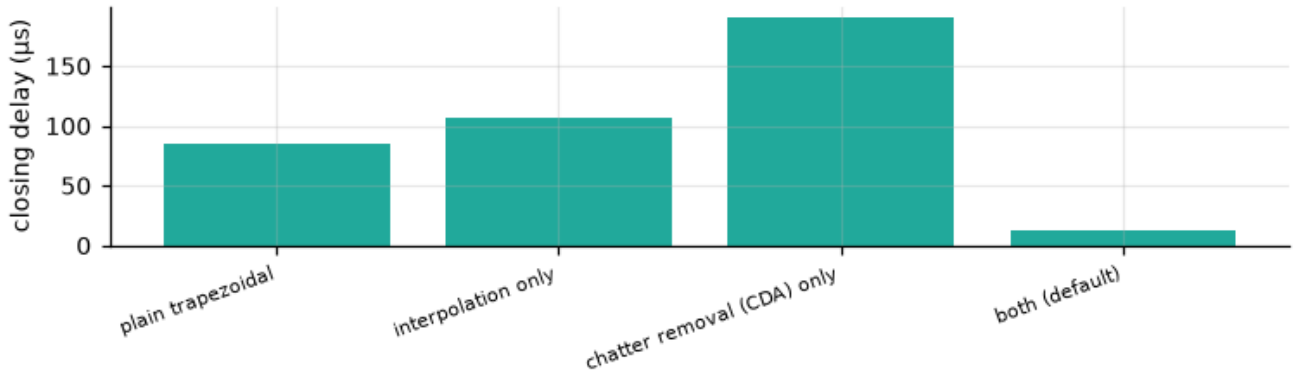
- **Code = method:** every current equals the ω_{trap} phasor to 2.36×10^{-12}.
- **Against the exact-ω phasor:** at most 2.03×10^{-5}, the reactance warping $(\omega\Delta t)^2/12 = 2.06\times10^{-5}$.

3. Switches and faults (`SW`, `F`)

3.1 Switch closing between time points

Model: a switch is 10 mΩ closed, 1 MΩ open; G is refactorised when it changes. Interpolation moves the solution back to the switching instant; CDA adds two backward-Euler half steps after it (docs/switching.md). **Test** (`demo_interpolation`): 10 V DC closes onto R = 1 Ω, L = 1 mH (τ = 1 ms) at 1.02 ms, Δt = 200 µs, so the instant is 0.18 ms before a time point. **Theory:** $i = V/R'(1 - e^{-(t - 1.02\ \text{ms})R'/L})$; each run's effective closing instant is fitted over 2–4 ms. The **discrete reference** applies the event-loop algorithm by hand to this one loop.





options	mechanism estimate (µs)	discrete reference (µs)	fitted delay of the simulation (µs)	simulation vs discrete reference (pu)	max error vs exact (pu of V/R')
plain trapezoidal	85.1	85.1	85.1	3.59×10^{-16}	0.0745
interpolation only	107	107	107	2.24×10^{-16}	0.0837
chatter removal (CDA) only	189	190	190	1.01×10^{-11}	0.166
both (default)	11.3	12	12	1.01×10^{-11}	0.00784

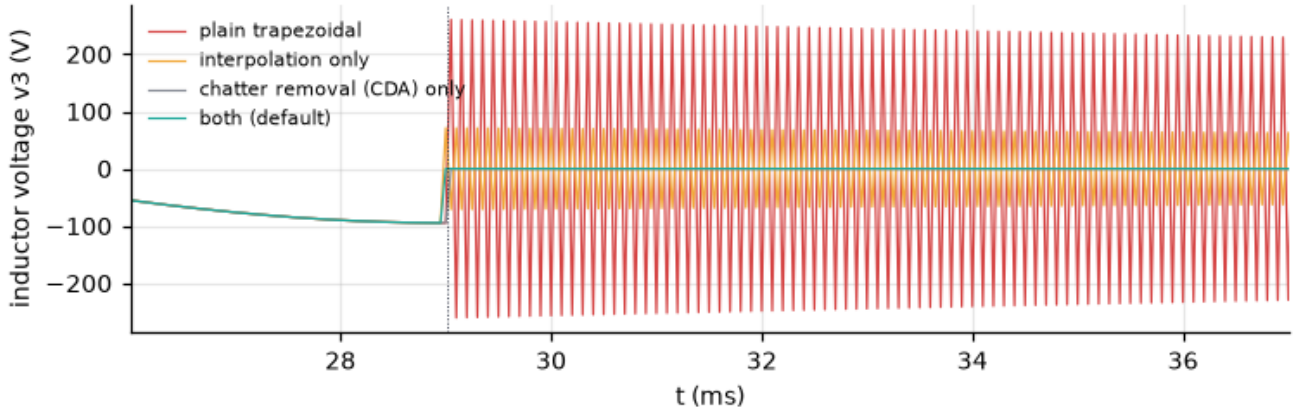
Where each delay comes from (first order, plus the second-order terms in the table's estimate):

options	mechanism	first-order delay
plain trapezoidal	closed for the whole step from 1.0 ms, plus the trapezoidal start-up Δt/2	80 µs
interpolation only	placed at 1.02 ms, but the trapezoidal step after it adds Δt/2	100 µs
CDA only	found at the end of the step (1.2 ms), then the half steps	180 µs + BE lag
both (default)	the half steps' own lag 9.5 µs (≈ $\Delta t^2/4\tau$) + interpolation back onto the grid 1.8 µs	second order

- **Code = method:** every option equals the hand-applied algorithm to round-off.
- **Default:** about 12 µs, 6 % of Δt. Interpolation and CDA belong together.

3.2 Breaker opening at a current zero (`SW … czero`)

Test (`demo_breaker`): 100 V, 50 Hz into R = 1 Ω and L = 10 mH; open command at 20.1 ms. **Theory:** the current zero is at t_z = 29.0099 ms (φ = 72.18°); afterwards only the 1 MΩ leakage flows, so the inductor voltage should be about 3.14×10^{-4} V.

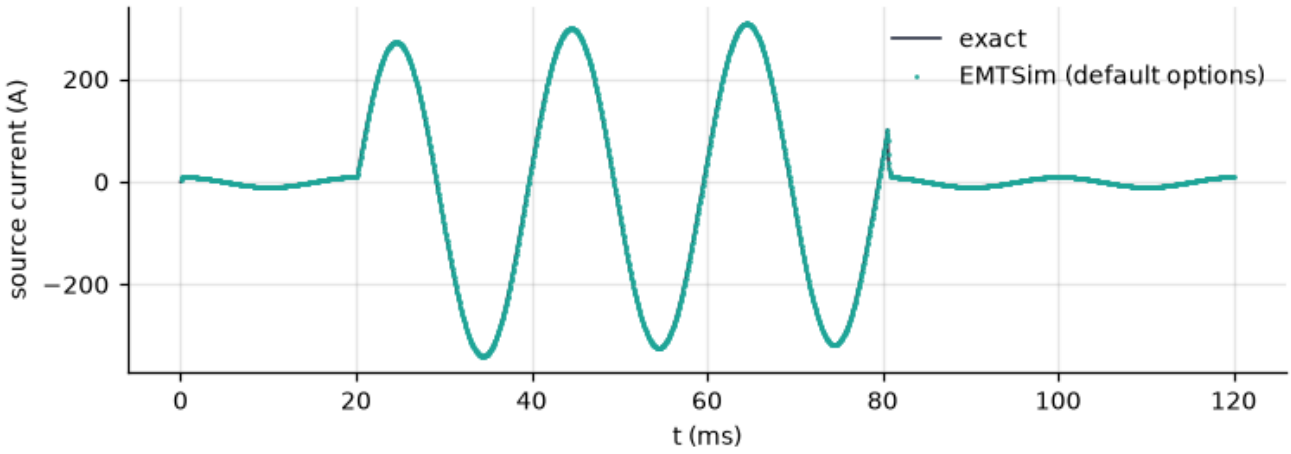


options	max |v_L| after opening (V)	max |i| after opening (pu of peak)
plain trapezoidal	260	1.18×10^{-5}
interpolation only	71.1	5.61×10^{-6}
chatter removal (CDA) only	0.0642	3.30×10^{-6}
both (default)	3.25×10^{-4}	3.30×10^{-6}

- **Without interpolation:** a small current is chopped at the next time point, and the trapezoidal rule turns it into a sustained ±V chatter.
- **CDA only:** the chatter is damped in two half steps, but the chop leaves a spike.
- **Both (default):** the breaker opens at the exact zero; afterwards only the leakage level remains.

3.3 Fault with a DC offset (`F`)

Model: a fault is a switch (10 mΩ on, 1 MΩ off) applied and cleared at given times. **Test:** 1000 V, 50 Hz behind 0.5 Ω + 10 mH into 100 Ω; a fault to ground at 20.1234 ms, cleared at 80.5678 ms (both off the 50 µs grid). **Theory:** one inductor, so the current is piecewise analytical, $i = i_{ss,k} + (i(t_k) - i_{ss,k}(t_k))\, e^{-(t - t_k)R_k/L}$, with the fault's 10 mΩ included.



options	theory: peak (A)	simulated peak (A)	max error during the fault (pu of the symmetrical peak)	max error in the 1 ms after clearing (pu)
plain trapezoidal	339	339	5.08×10^{-4}	0.0239
interpolation only	339	341	0.0079	0.0497
chatter removal (CDA) only	339	341	0.00841	0.0904
both (default)	339	339	2.91×10^{-5}	0.0105

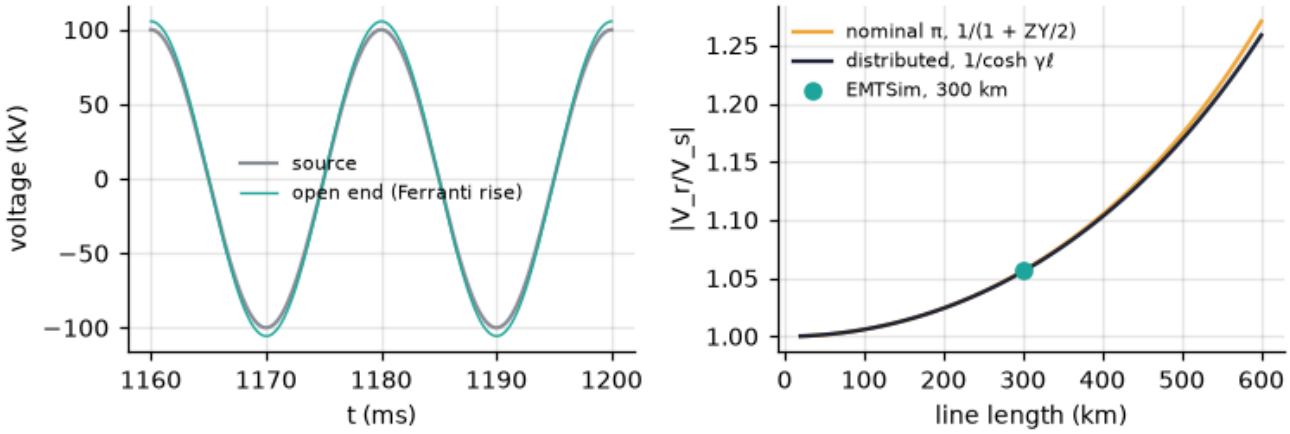
- **During the fault (default):** the asymmetric peak and waveform agree to 2.91×10^{-5} of the peak (trapezoidal error at $\omega\Delta t = 0.016$ plus the half steps). Without interpolation the DC offset is wrong and the error is about 20 times larger.
- **After clearing:** the load circuit's time constant is 0.1 ms, only two steps, so the first samples are about 1 % off: a resolution limit of Δt, not a switching error (Δt = 10 µs cuts it about 25-fold).

4. Lines and coupled branches (`PI`, `PI3`, `RL`, `RL3`, `TL`, `TL3`)

`RL` is the one-phase case of `RL3` and behaves as the inductor of chapter 1 with a series resistance. It is covered by sections 1.3, 2.2 and 4.2.

4.1 Nominal π line (`PI`)

Model: a series R–L branch with half the capacitance at each end. **Test:** a 300 km line (R = 9 Ω, L = 0.3 H, C = 3.6 µF in total) from an ideal 100 kV, 50 Hz source, far end open, Δt = 50 µs, 1.2 s (18 time constants of its 216 Hz resonance). **Theory:** nominal π 1/(1 + ZY/2) at ω and ω_{trap}; distributed line 1/cosh γℓ.

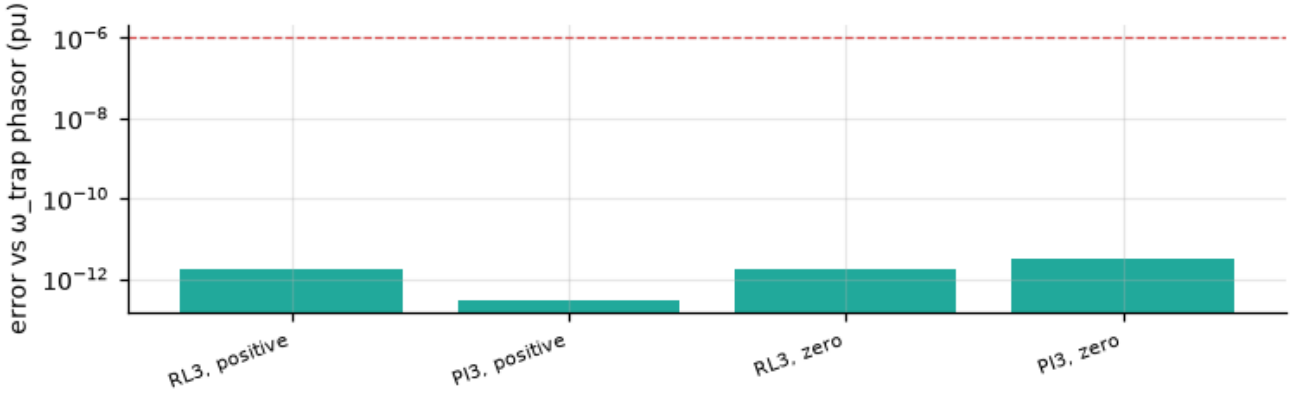


reference	|V_r/V_s|	angle (°)	simulated vs this reference (pu)
nominal π at ω_trap (the discretised model)	1.06	-0.308	1.74×10^{-9}
nominal π at ω	1.06	-0.308	2.32×10^{-6}
distributed line, 1/cosh(γℓ)	1.06	-0.302	5.05×10^{-4}
simulated	1.06	-0.308	0

- **Code = method:** equal to the nominal π at ω_{trap} to 1.74×10^{-9} (the line's own transient, decayed to e^{-18}).
- **Model simplification:** 5.03×10^{-4} from the distributed line at 50 Hz, growing as $(ZY)^2/24$ (the fourth power of length, right). For travelling waves use `TL`, or several π sections.

4.2 Coupled three-phase branches from sequence data (`RL3`, `PI3`)

Model: phase matrices from sequence values ($Z_s = (Z_0 + 2Z_1)/3$, $Z_m = (Z_0 - Z_1)/3$, and the same for C), discretised as coupled blocks. **Test:** Z_1 = 2 Ω + 50 mH, C_1 = 0.6 µF; Z_0 = 6 Ω + 150 mH, C_0 = 0.35 µF; positive-sequence or zero-sequence (three in-phase) 1000 V sources; `RL3` shorted, `PI3` open. **Theory:** each sequence sees only its own impedance: I = E/Z (`RL3`), V_r = E/(1 + ZY/2) (`PI3`).

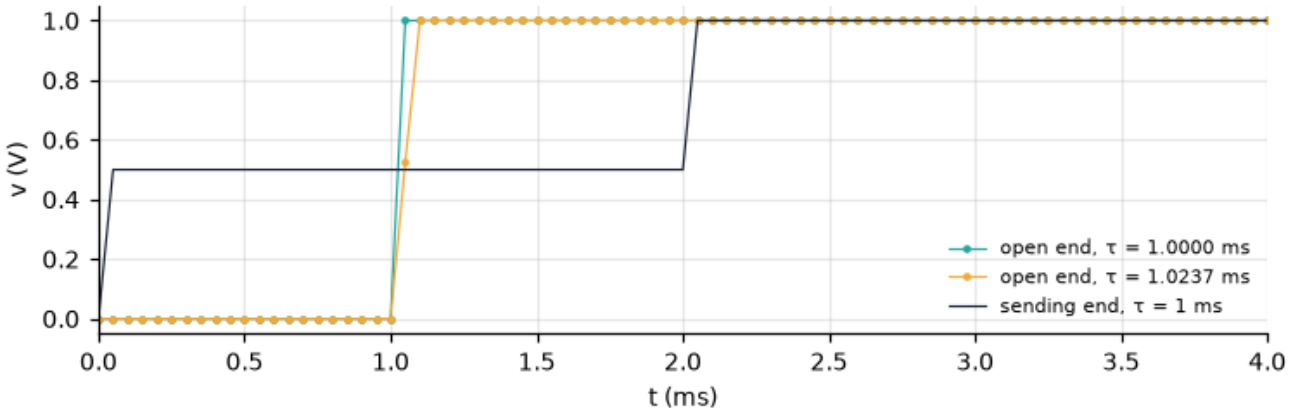


element	sequence	quantity	exact-ω value	simulated	vs ω_trap phasor (pu)
RL3, far end shorted	positive	|I_a| (A)	63.2	63.2	1.75×10^{-12}
PI3, far end open	positive	|V_r,a| (V)	1e+03	1e+03	2.97×10^{-13}
RL3, far end shorted	zero	|I_a| (A)	21.1	21.1	1.74×10^{-12}
PI3, far end open	zero	|V_r,a| (V)	1e+03	1e+03	3.16×10^{-12}

- **Decoupled sequences:** each sees only its own impedance, to 3.16×10^{-12} (the dashed line is the check's limit).

4.3 Bergeron travelling-wave line (`TL`)

Model: each end is a conductance 1/Z (Z = Z_c + R/4) with a history source from the other end one travel time τ earlier, read by linear interpolation (τ need not be a whole number of steps); losses lumped R/4, R/2, R/4. **Test (lossless):** 1 V DC behind R = Z_c = 400 Ω (matched) into an open line, Δt = 50 µs, τ = 1 ms (whole steps) and 1.0237 ms (not). **Theory:** 0.5 V enters, doubles to 1 V at the open end at τ, returns at 2τ and is absorbed; the reference resamples the far end's samples as the model does.



travel time	far end: 50 % arrival − τ (µs)	sending end: 75 % point − 2τ (µs)	sending end vs naive ramp delayed 2τ (V)	max |v − resampled reference| (V)
1.0000 ms (whole number of steps)	25	25	2.22×10^{-15}	2.22×10^{-16}
1.0237 ms (20.474 steps)	23.8	25	0.112	2.22×10^{-16}

- **Code = method:** both lines equal the reference to 2.22×10^{-16}.
- **Arrivals** are Δt/2 after the ideal time: the start-up ramp, not the line.
- **τ not a whole number of steps:** every traversal is the filter $(1-\alpha)+\alpha e^{-j\omega\Delta t}$ (α = frac(τ/Δt) = 0.474): DC gain 1, attenuation $\alpha(1-\alpha)(\omega\Delta t)^2/2$, i.e. 0.12 % at 1 kHz and 12 % at 10 kHz per traversal. The reflected front departs from the naive ramp by 0.112 V. Choose Δt so τ/Δt is whole where it matters.

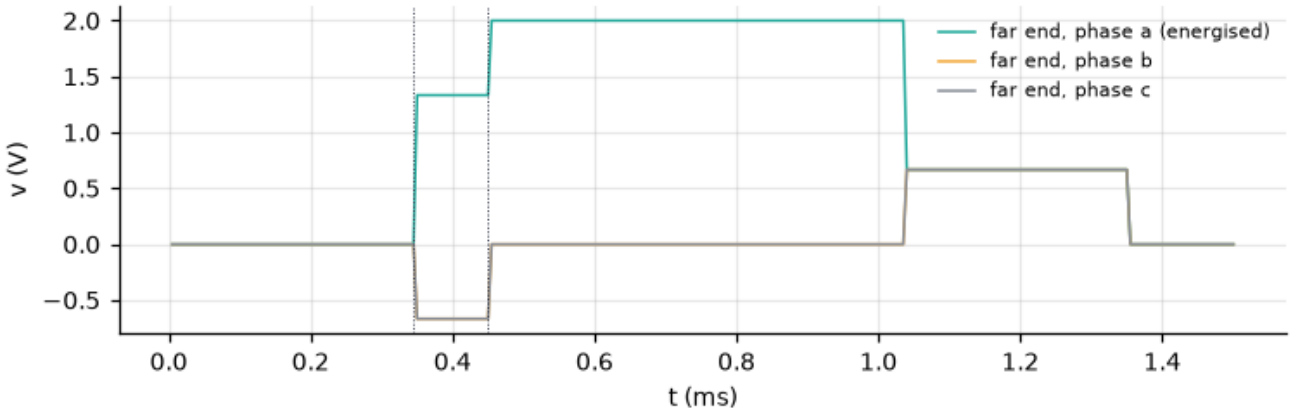
Test (lossy): the same with R = 0.2Z_c and τ = 1 ms. **Theory:** Bergeron gives an open-end plateau of $\frac{1}{2}(1+h)^2 v_k$, h = (Z_c − R/4)/(Z_c + R/4); a distributed line attenuates a front by $e^{-R/2Z_c}$.

reference	open-end voltage after the first arrival (V)
Bergeron with R/4, R/2, R/4 (the model)	0.929
distributed line, front $e^{-R/2Zc}$	0.905
simulated	0.929

- **Code = method:** the plateau equals the Bergeron value to round-off; the final value is 1.000000 V (theory 1).
- **Model simplification:** 2.7 % above the distributed line: lumped losses need R ≪ Z_c.

4.4 Three-phase Bergeron line (`TL3`)

Model: a transposed line, decoupled by the Clarke transformation into a zero mode (Z_0, τ_0) and two aerial modes (Z_1, τ_1). **Test:** Z_1 = 300 Ω, τ_1 = 0.345 ms; Z_0 = 600 Ω, τ_0 = 0.45 ms; phase a energised by a 1 V step, b and c grounded, far end open, Δt = 5 µs. **Theory:** the modal entry T·(1, 0, 0) doubles at the open end: phase a gets 4/3 at τ_1 and 2/3 more at τ_0 (total 2); b and c get −2/3 at τ_1, back to 0 at τ_0.

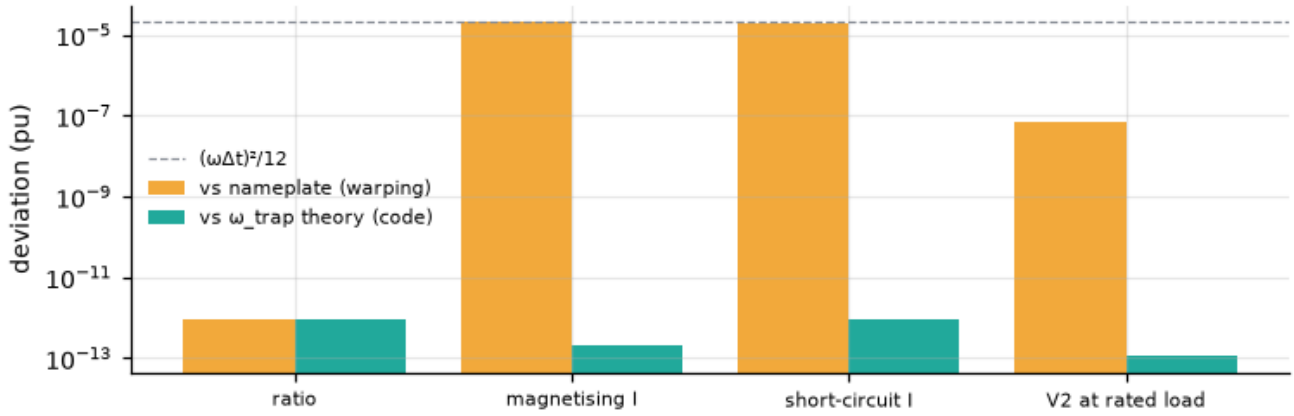


- **All three phases** equal the modal prediction to 4.44×10^{-16}: transformation, travel times and doubling.

5. Transformers (`XF`, `XF3`, `LSAT`)

5.1 Single-phase transformer (`XF`): ratio, leakage and magnetising branch

Model: an ideal ratio n = V_1/V_2, series r and x referred to the primary, and $L_m = V_1^2/(\omega S \cdot im/100)$ across the primary (two-slope with `knee`, section 5.2). **Test:** 1 MVA, 11/0.4 kV, x = 0.06, r = 0.005 pu, im = 1 %, rated voltage, secondary open, shorted or at rated load, Δt = 50 µs. **Theory** (at ω_{trap}): $V_2 = V_1/n$ open, magnetising current $im \cdot \omega/\omega_{trap}$ %, short circuit $1/|r + jx|$ pu.

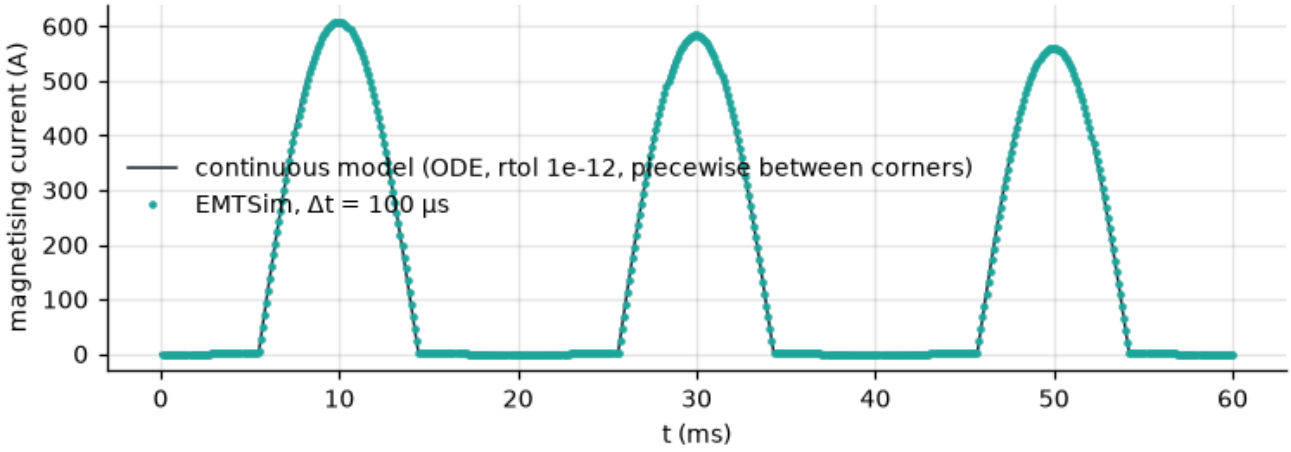


test	nameplate / exact-ω	theory at ω_trap	simulated	simulated vs ω_trap theory (pu)
open circuit: V1/V2	27.5	27.5	27.5	9.13×10^{-13}
open circuit: magnetising current (% of rated)	1	1	1	2.17×10^{-13}
short circuit: primary current (pu)	16.6	16.6	16.6	9.46×10^{-13}
rated resistive load: V2 (pu of rated)	0.993	0.993	0.993	1.13×10^{-13}

- **Code = method:** every quantity equals the ω_{trap} phasor to 9.46×10^{-13}.
- **Against the nameplate:** the currents differ by the warping $(\omega\Delta t)^2/12 = 2.06\times10^{-5}$; the open-circuit ratio is exact (the magnetising branch is on the source side).

5.2 Saturation and inrush (`XF … knee= lsat=`, `LSAT`)

Model: with `knee` the magnetising branch is a two-slope flux–current curve (L_m to knee·λ_R, then L_{sat}), flux by the trapezoidal rule, the segment chosen for the end of the step and the step re-solved until it holds (corners are not located within the step). **Test:** the 5.1 transformer with knee = 1.15, L_{sat} = 0.15 pu, energised at a voltage zero through 0.5 Ω + 10 mH, secondary open, Δt = 200 … 12.5 µs. **Reference:** $d\lambda/dt \cdot (1 + L_{src} f'(\lambda)) = v - R_{src} f(\lambda)$, integrated by 8th-order Runge–Kutta, restarted at every knee crossing. The textbook flux doubling (no source impedance) would give 730 A; with it the peak is 609.2 A.

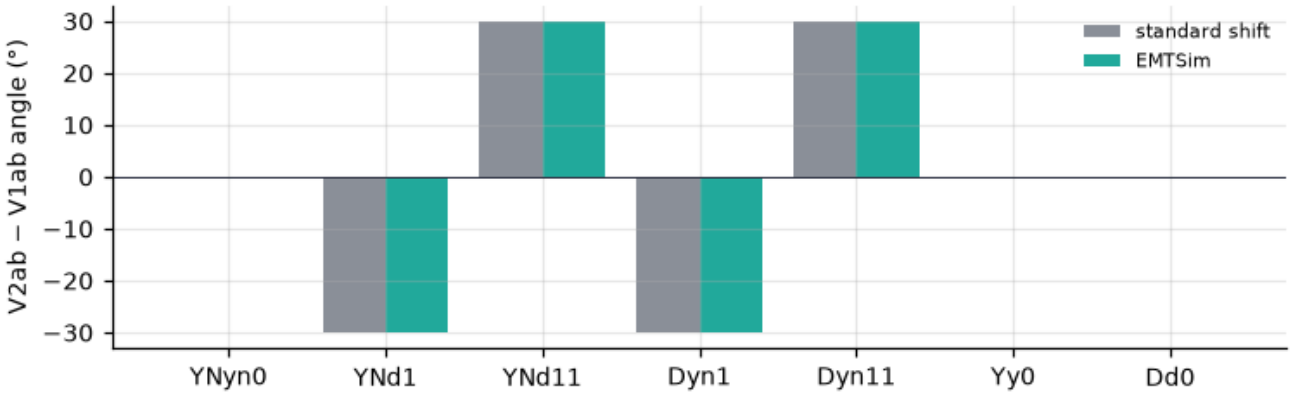


Δt	first peak (A)	peak vs continuous (pu)	max error over 60 ms (pu of peak)	order
200 µs	609	-0.001	7.72×10^{-4}	–
100 µs	609	-2.76×10^{-4}	1.92×10^{-4}	2.01
50 µs	609	-5.72×10^{-5}	4.82×10^{-5}	1.99
25 µs	609	-2.10×10^{-5}	1.20×10^{-5}	2.01
12.5 µs	609	-3.01×10^{-6}	3.01×10^{-6}	1.99

- **Peak** within 5.72×10^{-5} of the continuous model at 50 µs; the whole run converges at order 1.99, though corners are not located (the current is continuous in the flux, so a corner costs $O(\Delta t^2)$ per step).
- **Guidance:** 200 µs already gives the inrush peak to 0.1 %.

5.3 Three-phase transformer vector groups (XF3)

Model: three single-phase units (S/3 each) in star, grounded star or delta; the delta orientation sets the clock number (1: secondary lags 30°, 11: leads 30°); a delta gets 100 MΩ to ground. **Test:** 60 MVA, 132/33 kV, x = 0.12 pu, rated voltage, secondary open (10 MΩ); shift and ratio from V_{ab}.



connection	expected shift (°)	measured shift (°)	rated ratio V1/V2	measured ratio
YNyn0	0	-1.25×10^{-5}	4	4
YNd1	-30	-30	4	4
YNd11	30	30	4	4
Dyn1	-30	-30	4	4
Dyn11	30	30	4	4
Yy0	0	-1.25×10^{-5}	4	4
Dd0	0	-1.37×10^{-5}	4	4

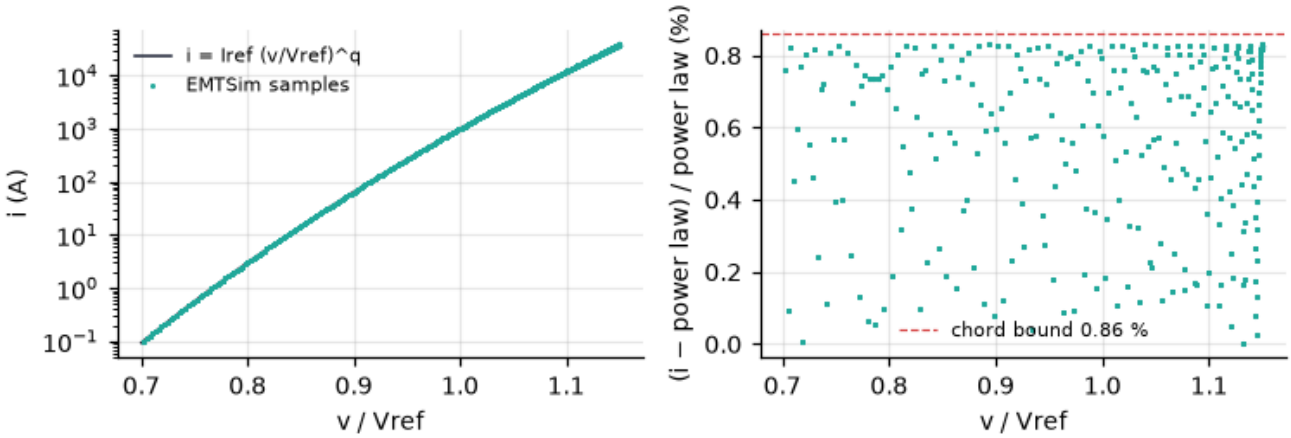
- **Every connection** gives its standard shift and rated ratio to 4.58×10^{-7} (the remainder: magnetising current through the leakage).

6. Non-linear elements and valves (`MOV`, `D`, `THY`)

`LSAT` is verified in section 5.2 as the transformer's magnetising branch.

6.1 Metal-oxide surge arrester (`MOV`)

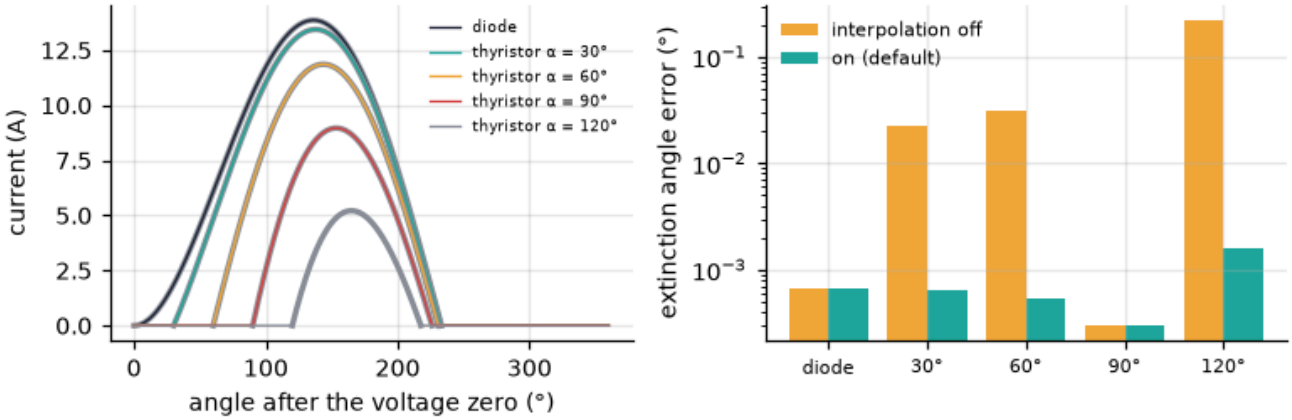
Model: $i = I_{ref}(|v|/V_{ref})^q$ as a piecewise-linear conductance, break points from 0.5 to 2.5 V_{ref} with the current growing 30 % per segment; the step is re-solved until the segment holds. **Test:** 1.15 V_{ref} peak, 50 Hz, directly across a 100 kV, 1 kA, q = 26 arrester, Δt = 10 µs. **Theory:** between break points the chord over-reads the power law by at most $\max_u (1 + 0.3u)/1.3^u - 1 = 0.863$ % and never under-reads (convex).



- **Code = method:** equal to the piecewise-linear curve to 6.69×10^{-15}.
- **Against the power law:** 0.004 % to 0.830 %, inside the chord bound: the current (and energy) is up to about 0.86 % high at a given voltage, the protective level 0.03 % low at a given current.

6.2 Diode and thyristor (`D`, `THY`): conduction, firing and extinction

Model: 1 mΩ on, 1 MΩ off; a diode turns on above v_f and off when its current reverses; a thyristor may turn on while (360ft + ref) mod 360 is within [α, α + width]. With interpolation, turn-on and turn-off are located inside the step, followed by CDA half steps. **Test:** a half-wave rectifier, 100 V, 50 Hz into R = 5 Ω, L = 20 mH (φ = 51.5°), Δt = 50 µs (0.9°); a diode and a thyristor at α = 30 … 120°; the fourth (periodic) cycle. **Theory:** $i(\psi) = (V_m/|Z|)[\sin(\psi - \varphi) - \sin(\alpha - \varphi)e^{-(\psi-\alpha)/\tan\varphi}]$, extinction β where i = 0, mean load voltage $[R/(R + R_{on})]V_m(\cos\alpha - \cos\beta)/2\pi$ (grey curves).



valve	β theory (°)	β, interpolation on (°)	β, interpolation off (°)	mean V theory (V)	mean V, interp on (V)	mean V, interp off (V)
diode	233.238	233.239	233.239	25.4356	25.4360	25.4339

valve	β theory (°)	β, interpolation on (°)	β, interpolation off (°)	mean V theory (V)	mean V, interp on (V)	mean V, interp off (V)
thyristor α = 30°	232.738	232.739	232.716	23.4148	23.4152	23.3347
thyristor α = 60°	230.690	230.691	230.659	18.0368	18.0366	17.9692
thyristor α = 90°	226.087	226.087	226.087	11.0363	11.0354	11.0337
thyristor α = 120°	217.608	217.606	217.386	4.6497	4.6487	4.5414

- **Interpolation on (default):** β within 0.002° (or % of V_m for the mean voltage): the R–L load's trapezoidal error and the half steps.
- **Interpolation off:** the thyristor fires and the diode stops at the next time point, up to one step late: worst 0.22°.

7. Machines (`SM`, `IM`)

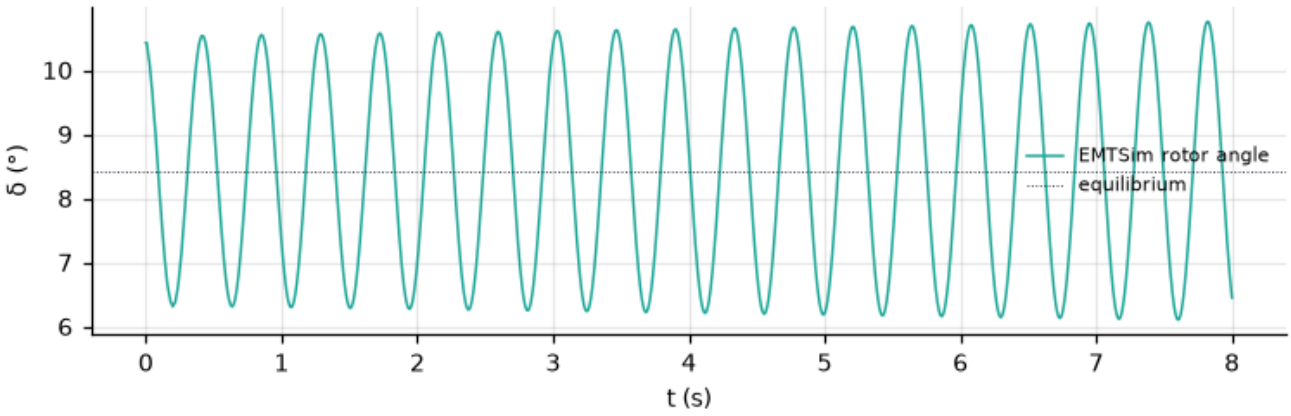
7.1 Synchronous machine (`SM`), classical model

Model: EMF $E \cdot V_{pk} \cos(\omega_s t + \delta - 2\pi k/3)$ behind x_d'' and r_a (a trapezoidal three-phase R–L, so stator transients are included); $2H\, d\omega/dt = P_m - P_e - D(\omega - 1)$, $d\delta/dt = \omega_s(\omega - 1)$, semi-implicit Euler. **Test 1 (steady state):** 100 MVA, 20 kV, $x_d'' = 0.3$, $r_a = 0.005$, E = 1.2 pu on an ideal 1 pu bus, P_m = 0.3 / 0.6 / 0.9 pu, D = 20, 6 s at 100 µs. **Theory:** δ from $\mathrm{Re}(Ee^{j\delta} \cdot \mathrm{conj}\ I) = P_m$, $I = (Ee^{j\delta} - 1)/(r_a + jX)$, at X and at $X\omega_{trap}/\omega$.

Pm (pu)	δ lossless (°)	δ exact (°)	δ at ω_trap (°)	δ simulated (°)	Pe simulated (pu)
0.3	4.3012	4.1085	4.1088	4.1088	0.300001
0.6	8.6269	8.4258	8.4265	8.4265	0.600000
0.9	13.0029	12.7863	12.7874	12.7874	0.900000

- **Load angle** equals the phasor solution at the trapezoidal reactance to 0.0000°, and $P_e = P_m$ to 5.51×10^{-7}.

Test 2 (small oscillation): P_m = 0.6 pu, D = 0, started 2° off, 8 s. **Theory:** the quasi-static swing gives $f_n = \sqrt{(\omega_s P_s/2H)}/2\pi$ = 2.2908 Hz, undamped; this model also has stator flux dynamics, whose four-state eigenvalue is $\sigma \pm j2\pi f = 0.01342 \pm j2\pi \cdot 2.2963$ ($0.00000 \pm j2\pi \cdot 2.2932$ with $r_a = 0$).

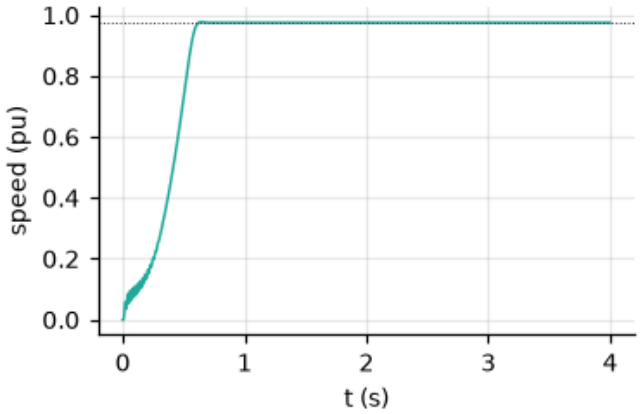


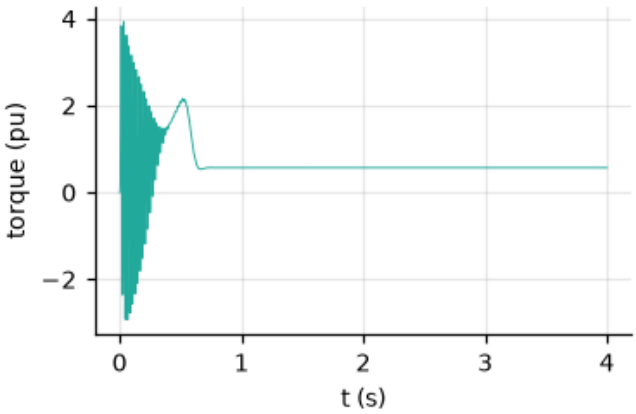
quantity	quasi-static	eigenvalue	EMTSim
frequency (Hz)	2.2908	2.2963	2.2959
growth σ (1/s)	0	0.01342	0.01342

- **Frequency** -0.014 % from the eigenvalue (the amplitude effect of a 2° swing); +0.22 % from the quasi-static formula, which omits the stator dynamics.
- **Slow growth** (2.137° → 2.333°): the eigenvalue's σ, from r_a with the stator transients; the same at 100, 50 and 25 µs, so not numerical. Use D (a few pu) to represent dampers.

7.2 Induction machine (`IM`)

Model: a voltage behind transient inductance L′ in the stator frame; rotor flux $d\psi_r/dt = -(R_r/L_r)(\psi_r - L_m i) + j\omega_r\psi_r$ (trapezoidal); the network uses the e′ of the previous step (the machine updates after the network). **Test:** 2 MVA, 11 kV, 4 poles, r_s = 0.01, x_{ls} = 0.1, r_r = 0.04, x_{lr} = 0.1, x_m = 3 pu, H = 0.3 s, T_{load} = 0.6 pu, started direct on line, 4 s, three time steps. **Four closed-form steady states:** (1) the T-equivalent circuit, s = 0.025016; (2) the model's continuous equations, s = 0.025016 (= (1)); (3) the discretised equations at ω_{trap}; (4) as (3) with e′ one step late, which is what the code solves.





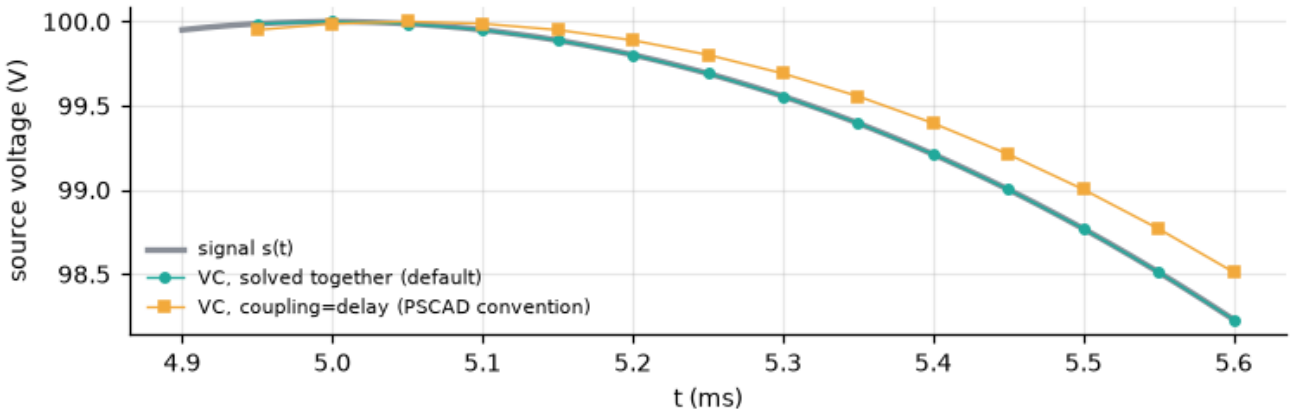
Δt	(2) continuous	(3) warping only	(4) warping + EMF delay	simulated	simulated vs (4) (pu)	simulated vs (2) (pu)
100 µs	0.025016	0.024942	0.024772	0.024772	7.68×10^{-11}	0.00978
50 µs	0.025016	0.024998	0.024912	0.024912	8.40×10^{-11}	0.00414
25 µs	0.025016	0.025011	0.024969	0.024969	8.35×10^{-11}	0.00189

- **Code = method:** the slip equals (4) at every Δt.
- **Warping** (3) − (2): 0.3 % of the slip at 100 µs, order 2. **The EMF delay** (4) − (3): first order and dominant, 0.98 % at 100 µs (observed order 1.14); in absolute terms the speed is off by 2.4e-04 pu.
- **Guidance:** for slip and loss studies use Δt ≤ 25 µs. (Solving the machine with the network, as is now done for the controls in section 8.1, would remove the delay.)

8. Controlled elements and meters (`VC`, `IC`, `SWC`, `THYC`, `VM`, `METER`)

8.1 Controlled sources and meters (`VC`, `IC`, `VM`, `METER`)

Model: meters read the solution of the same step. Controlled sources (`VC`, `IC`, `VC3`, `IC3`, a machine's `ef=` / `pm=`) are **solved together** with the network: when a driving signal comes out different from the value used, the step is solved again from its start with the new values, until they agree to 10^{-6} of the signal's peak (a fixed-point iteration; the first guess extrapolates the last two steps; a step that does not settle keeps its first solution and the run warns). `.options coupling=delay` keeps the one-step delay of TACS/PSCAD. **Test:** `SINE` (100 sin ωt, 50 Hz) drives a `VC` and an `IC` into 10 Ω; a `VM` and a `METER` read them; Δt = 50 µs; with and without `coupling=delay`.



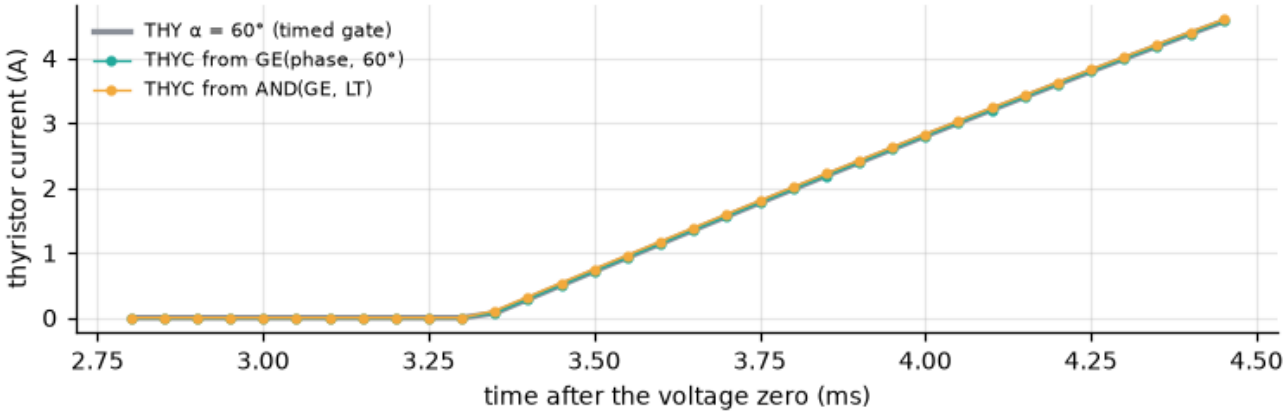
element	max deviation (pu of amplitude)
VC: v = s(t)	1.75×10^{-15}
IC: i = s(t)	1.75×10^{-15}
VM: signal = node voltage in the same step	0
METER: signal = recorded current in the same step	0
VC with `coupling=delay`: v = s(t − Δt)	1.42×10^{-16}
IC with `coupling=delay`: i = s(t − Δt)	2.27×10^{-16}

coupling	phase lag at 50 Hz	order of a closed loop (`tests/test_emtsim.cpp`)
solved together (default)	-1.4e-14°	1.9
`coupling=delay` (PSCAD)	0.900° (ωΔt = 0.900°)	1.0

- **Solved together:** the sources equal the signal of the same step to round-off. A one-step delay is a first-order error in any closed loop (18° at 1 kHz and 50 µs); removing it makes the library's inverter demos 2 to 60 times closer to an 8-times-finer run, for 50–70 % more run time.
- **Meters** read the same step with no delay.

8.2 Switches and thyristors driven by signals (`SWC`, `THYC`)

Model: closed (or fired) while the signal exceeds 0.5. With interpolation the element switches at the instant the signal changed: from a comparator (`GT LT GE LE`, also through one `NOT`), where its input difference crossed zero; from any other block, where the signal itself crosses 0.5. These are timed, not solved together. **Test 1:** the circuit of 3.1 closed by a timed `SW` and by a `SWC` from `GE g t 1.02e-3`. **Test 2:** the α = 60° rectifier of 6.2 fired by the timed gate, by `THYC` from `GE` on a phase signal, and by `AND` of two comparators (figure: the firing, zoomed).



test	interpolation	max |i − i_SW| (pu of V/R)
SWC from GE(TIME, 1.02 ms) vs timed SW	on	0
SWC from GE(TIME, 1.02 ms) vs timed SW	off	0

gate	β theory (°)	β simulated (°)	max |i − i_THY| (pu of peak)
THY α = 60° (timed gate)	230.690	230.691	0
THYC from GE(phase, 60°)	230.690	230.691	1.99×10^{-12}
THYC from AND(GE, LT)	230.690	230.706	0.00302

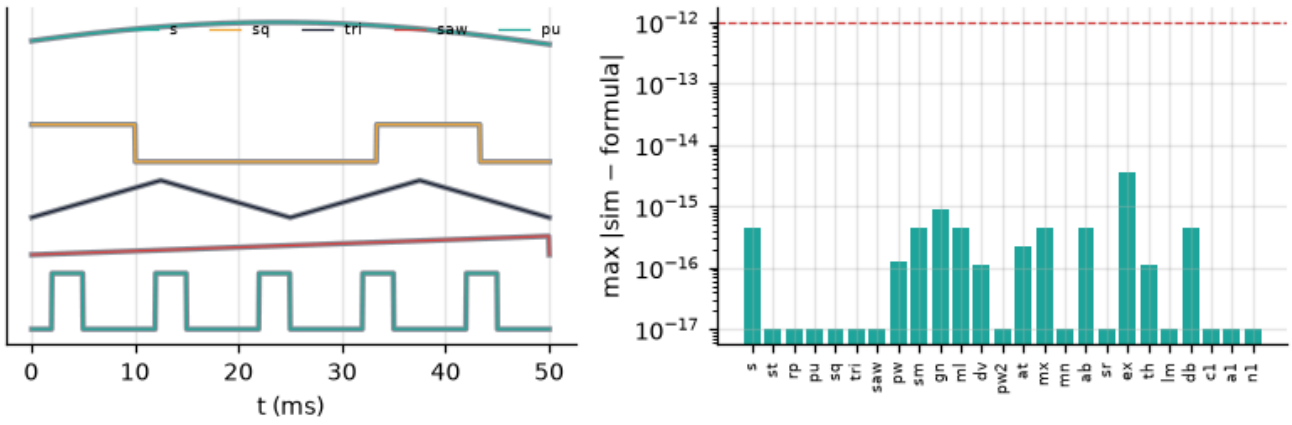
- **One comparator:** its crossing instant is exact; controlled elements equal the timed ones to round-off.
- **Through `AND`:** the output crosses 0.5 mid-step, so the firing is up to Δt/2 = 0.45° late: here 0.30 % of the peak current and 0.015° in β. For exact timing take the gate from one comparator.

9. Control blocks

Every block (docs/controls.md), verified by group.

9.1 Signal sources, mathematical functions, limiters and logic

Model: algebraic functions of time or of their inputs (docs/controls.md), with no discretisation; an edge at a time point switches when $t \geq t_{edge} - 10^{-6}\Delta t$. **Test:** 25 blocks every 50 µs for 50 ms against their formulas in numpy (left: five of them, grey = formula; right: every block, dashed = 10^{-12}).

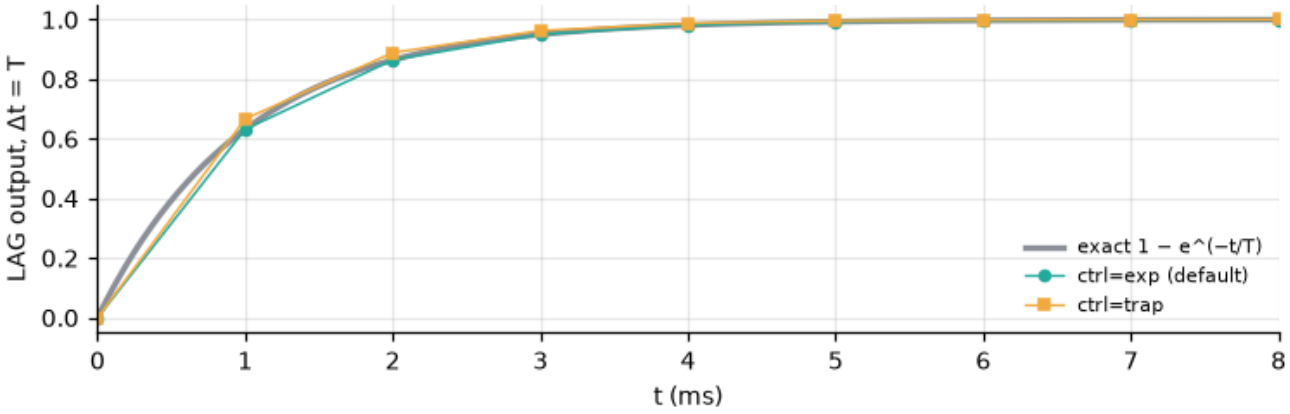


- **All exact** to 3.55×10^{-15} (round-off).

9.2 Linear blocks: exponential form and trapezoidal rule

Model: `LAG`, `LEADLAG`, `WASHOUT`, `DERIVF`, `LP2`, `BP2`, `NOTCH`, `TF`, `SS`, `BUTTER`, `CHEBY` and `PADE` share $x(t) = Mx(t - \Delta t) + N_0u(t - \Delta t) + N_1u(t)$: with `ctrl=exp` (default) from a matrix exponential, exact for an input linear between samples; with `ctrl=trap` the bilinear map. No start-up lag (the first evaluation sets $u(t - \Delta t) = u(0)$). **Tests:** `LAG` steps at $\Delta t/T = 0.1$ and 1 (trapezoidal: $y_k = 1 - \rho^k$); a 2nd-order `TF` with a zero against `scipy.signal.lsim`; `LP2` driven at its natural frequency (exact gain $1/2\zeta$; trapezoidal

$G(j\omega_{trap})|$).



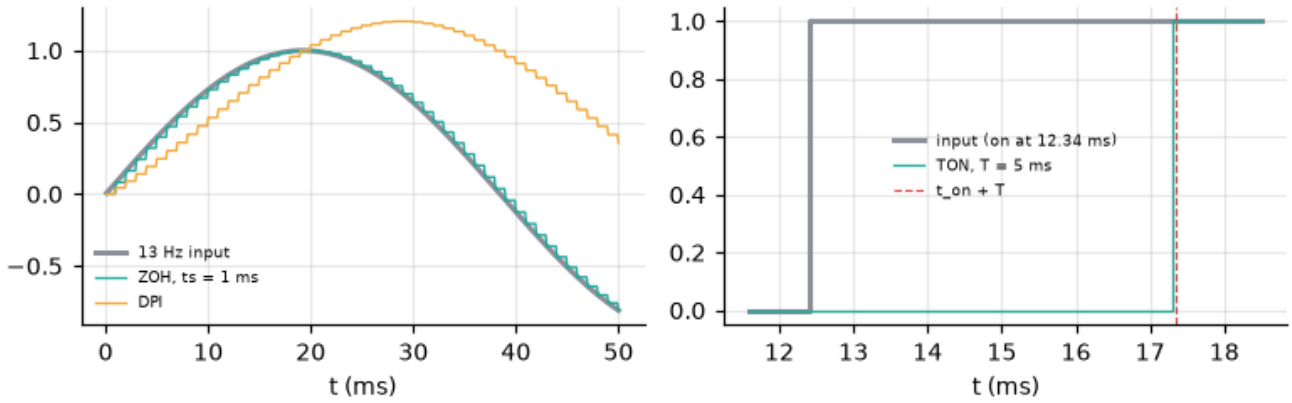
test	ctrl	theory	max error / relative error
LAG step, $\Delta t/T = 0.1$	exp	exact	2.44×10^{-15}
LAG step, $\Delta t/T = 0.1$	trap	max $\lvert\rho^k - e^{(-k\Delta t/T)}\rvert = 3.07\times10^{-4}$	3.07×10^{-4}
LAG step, $\Delta t/T = 1$	exp	exact	2.00×10^{-15}
LAG step, $\Delta t/T = 1$	trap	max $\lvert\rho^k - e^{(-k\Delta t/T)}\rvert = 0.0345$	0.0345

test	ctrl	theory	max error / relative error
TF (s + 200)/($10^{-4}s^2$ + 0.02s + 1), Δt = 200 µs	exp	lsim, piecewise-linear input	6.48×10^{-14}
same	trap	lsim, piecewise-linear input	2.59×10^{-5}
LP2 at f = f_n, |G| = 1/2ζ = 1.6667	exp	1.664475 (G times the triangle-hold gain)	0.00132
same	trap	1.6645 (bilinear: s → jω_trap)	0.00133

- `exp`: a step is exact at any Δt/T; the `TF` equals `lsim` to 6.48×10^{-14}; for a sinusoid the input's triangle hold gives $[\sin(\omega\Delta t/2)/(\omega\Delta t/2)]^2 \approx 1 - (\omega\Delta t)^2/12$ (0.00132 at the resonance), with the poles exact.
- `trap`: exactly the predicted errors: $1 - \rho^k$ (wrong by 0.0345 at Δt = T, Watson & Arrillaga fig. 8.5) and the warped resonance. Keep `exp`; `trap` is for comparing with programs that use it.

9.3 Timers and sampled blocks

Model: timers (`TON`, `TOFF`, `MONO`) count in whole steps from the first sample with the input on; sampled blocks (`ZOH`, `ZDELAY`, `DPI`, `DTF`, `FIR`) act at multiples of ts and hold. **Test:** `TON` with T = 5 ms, its input switched on at 12.34 ms (between 100 µs points); `ZOH` and `DPI` at ts = 1 ms on a 13 Hz sine.

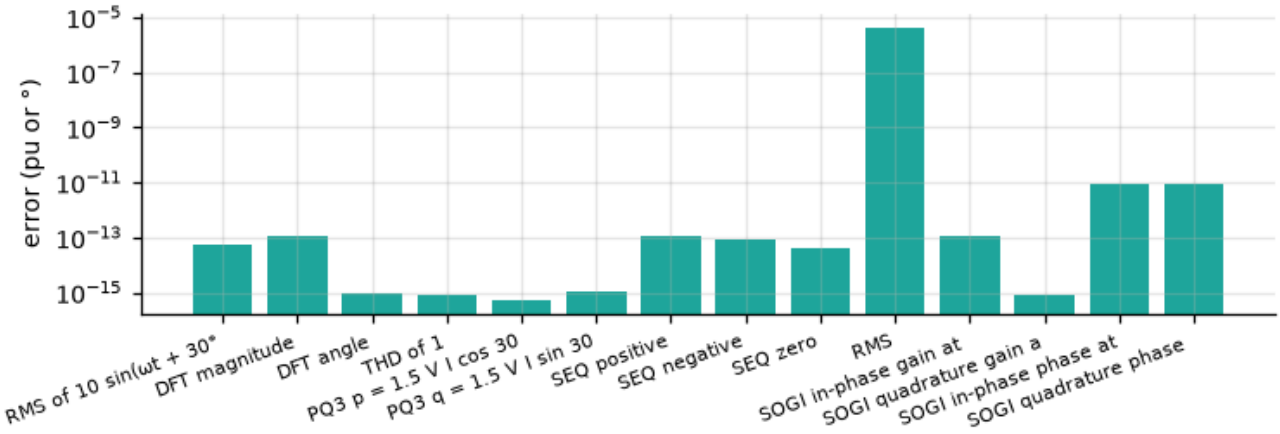


block	theory	simulated	comment
TON: output on at	t_prev + T = 0.01730 s (input on at 0.01234 s, previous time point 0.01230 s)	0.01730 s	-40.0 µs from t_on + T
ZOH (ts = 1 ms)	u(k ts) held	max deviation 2.03×10^{-14}	exact
DPI (ts = 1 ms)	kp e[n] + Σ ki ts e[k]	max deviation 1.52×10^{-14}	the documented difference equation

- **Timer:** on at t_{prev} + T, -40.0 µs from t_{on} + T (between 0 and one step early). For exact timing use a comparator on `TIME` (section 8.2).
- **Sampled blocks** equal their difference equations exactly.

9.4 Measurement blocks: RMS, DFT, THD, p–q, sequence components, SOGI

Model: `RMS`, `DFT`, `THD` integrate one period (trapezoidal on the samples, the window edge interpolated); `PQ3` is algebraic; `SEQ` is a one-period DFT per phase; `SOGI` is trapezoidal, so it evaluates $D(s) = k\omega s/(s^2 + k\omega s + \omega^2)$ at $j\omega_{trap}$. **Theory:** with a whole number of samples per cycle the windowed values are exact.

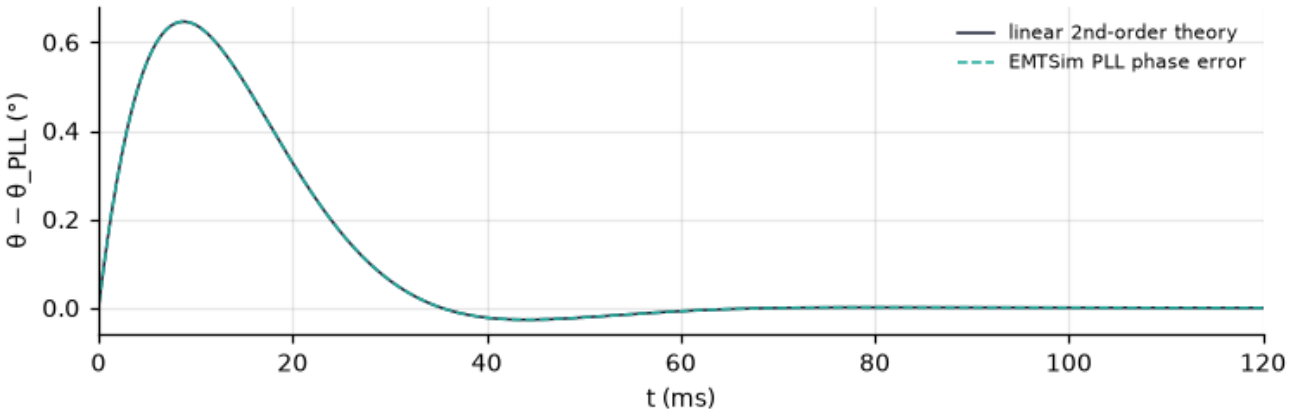


quantity	theory	simulated	error (pu or °)
RMS of 10 sin(ωt + 30°)	7.07	7.07	5.31×10^{-14}
DFT magnitude	10	10	1.05×10^{-13}
DFT angle (°, cosine reference: 30 − 90)	-60	-60	9.47×10^{-16}
THD of 1, 0.1 (3rd), 0.05 (5th)	0.112	0.112	7.91×10^{-16}
PQ3 p = 1.5 V I cos 30°	1.3e+03	1.3e+03	5.25×10^{-16}
PQ3 q = 1.5 V I sin 30°	750	750	1.06×10^{-15}
SEQ positive	10	10	1.06×10^{-13}
SEQ negative	2	2	8.13×10^{-14}
SEQ zero	0	3.90×10^{-14}	3.90×10^{-14}
RMS, Δt = 47 µs (425.5 samples per cycle)	7.07	ripple $\pm4.32\times10^{-6}$ pu	4.32×10^{-6}
SOGI in-phase gain at f0 (exact 1)	1.000000000 (bilinear)	1.000000000	1.11×10^{-13}
SOGI quadrature gain at f0 (exact 1)	0.999979438 (bilinear)	0.999979438	7.77×10^{-16}
SOGI in-phase phase at f0 (exact 0°)	-0.001666° (bilinear)	-0.001666°	8.92×10^{-12}
SOGI quadrature phase at f0 (exact −90°)	-90.001666° (bilinear)	-90.001666°	9.15×10^{-12}

- **400 samples per cycle:** RMS, DFT, THD, p, q, sequences exact to 1.06×10^{-13}.
- **47 µs (not a whole number per cycle):** the RMS ripples $\pm4.32\times10^{-6}$ at 2f; choose Δt so 1/(fΔt) is whole.
- **SOGI:** equal to the bilinear prediction: -0.00167° in phase at 50 µs, quadrature gain ω/ω_{trap} (warping).

9.5 Phase-locked loop (`PLL`)

Model: an SRF PLL on the Clarke components: $e = \sin(\theta - \hat{\theta})$ (evaluated at the angle predicted from the last frequency, so a locked PLL has no lag), $\omega = \omega_0 + k_p e + k_i \int e$, $\hat{\theta} = \int \omega$ (trapezoidal). **Test:** a 50.5 Hz input, f_0 = 50 Hz, k_p = 180, k_i = 16000, Δt = 50 µs. **Theory:** $\ddot{e} + k_p\dot{e} + k_i e = 0$, $e(t) = (\Delta\omega/\omega_d)e^{-\zeta\omega_n t}\sin\omega_d t$ (ω_n = 126.5 rad/s, ζ = 0.712, peak 0.647°); type 2, so zero steady error.



- **Transient** within 0.003 % of the linear theory's peak (the sin e ≈ e nonlinearity, e ≤ 0.011 rad).
- **Steady state:** 50.500000 Hz, the input frequency.

10. Adding a component

A component is a new element type in its own files; EMTSim finds it by its netlist keyword. Two come with it:

keyword	netlist line	what it is
`I`	`I a b peak angle freq`	sinusoidal current source from a into b (freq = 0: DC)
`CR`	`CR a b C R`	capacitor with a parallel leakage resistance, as one branch

part	file	needed for
C++ model	`src/components/NAME.cpp`	the solver, the tests, the browser engine
editor entry	`gui/components.js`	drawing it (two-terminal components)
check	`tests/test_emtsim.cpp`, a circuit in `circuits/`	proving it works

The build compiles every file in `src/components/`, the reader finds the keyword, and the editor lists the entries under *Added components*.

10.1 How an element looks to the solver

Each step, an element of k nodes is **i = g v + h**: v its terminal voltages, i the currents into it, g a k × k conductance (into G), h history currents from its past. For L or C the trapezoidal rule gives g = Δt/2L or 2C/Δt and h from the last step; a source has g = 0. Per step: `history(t)` (h), the network solve, `update(t, v)` (keep the result). `stamp()` gives g when G is built and when something switches.

10.2 Step by step: the lossy capacitor `CR`

1. **On paper:** $i_C(t) = (2C/\Delta t)v(t) - [(2C/\Delta t)v(t - \Delta t) + i_C(t - \Delta t)]$, so $g = 2C/\Delta t + 1/R$ and $h = -i_C(t - \Delta t) - (2C/\Delta t)v(t - \Delta t)$, with $i_C = i - v/R$.
2. **Base class:** a two-terminal branch derives from `TwoTerminal` (stamp from `Reff` = 1/g, keeps `Ilast`/`Vlast`, records, saves state); you set `Reff` and write `calcHistory`. Others derive from `Element` (as `I` does).
3. **The file** `src/components/lossy_capacitor.cpp`:

```
// CR  a b  C R                                                  # NAME
//
// Capacitor C with a leakage (or discharge) resistance R in parallel, as one branch. The trapezoidal rule
// gives the capacitor the conductance 2C/dt and a history current; R adds 1/R. Twin:
components/lossy_capacitor.py.

#include <cmath>
#include <stdexcept>

#include "component.h"

namespace {

class LossyCapacitor : public TwoTerminal {
public:
    LossyCapacitor(int a, int b, double C, double R) : TwoTerminal("CR", a, b), gc_(2.0 * C / config::Dt),
R_(R) {
        if (!(C > 0.0 && R > 0.0)) throw std::invalid_argument("CR: C and R must be positive");
        Reff = 1.0 / (gc_ + 1.0 / R);
    }

    // steady-state start: the admittance at w as the trapezoidal rule sees it
    cplx y(double w) const override { return cplx(1.0 / R_, gc_ * std::tan(0.5 * w * config::Dt)); }

protected:
    // Ilast and Vlast are the branch current and voltage of the last step (TwoTerminal keeps them)
    void calcHistory(double t) override {
        (void)t;
```

```
        const double ic = Ilast - Vlast / R_;   // the capacitor's share of the last current
        ihistory = config::euler ? -gc_ * Vlast : -ic - gc_ * Vlast;   // backward-Euler half step: no current
term
    }

private:
    double gc_, R_;
};

}  // namespace

EMTSIM_COMPONENT("CR", 2, [](ComponentLine& line) {
    line.add<LossyCapacitor>(line.node(0), line.node(1), line.number(0, "C"), line.number(1, "R"));
});
```

- `config::euler` marks chatter removal's backward-Euler half steps (same conductance, no current term).
- `y(w)` is the admittance as the trapezoidal rule sees it, for `init=steady` and `.scan`.
- `EMTSIM_COMPONENT(keyword, nodes, builder)` registers it; the builder uses `line.node(k)`, `line.number(k, "what"[, default])`, `line.key("key", default)`, `line.newNode()`, `line.add<T>(...)`.
- State beyond `Ilast` / `Vlast` goes into `saveState` / `loadState`, in the same order.

Build and run:

```
.\build.bat                                          # compiles src/components/*.cpp, runs the tests
.\build\emtsim.exe circuits\examples\test_components.net --chatter=cda
.\run.bat circuits\examples\test_components.net                          # and plot
```

It is then a netlist line like any other (`CR 3 0 200e-6 100 # BANK`), also inside modules. Rebuild the browser engine with `python gui/build_wasm.py`.

1. **Editor (optional):** an entry in `gui/components.js`:

```
    {
      key: 'CR', label: 'Capacitor with leakage resistance', short: 'Lossy C', prefix: 'C',
      params: [
        { key: 'C', label: 'Capacitance', unit: 'F', def: '100u' },
        { key: 'R', label: 'Parallel (leakage) resistance', unit: 'Ω', def: '10k' }],
      symbol: '<path d="M0 0 H36 M44 0 H80"/><path class="thick" d="M36 -13 V13 M44 -13 V13"/>' +
              '<path d="M26 0 V22 H32 M48 22 H54 V0"/><rect x="32" y="18" width="16" height="8" rx="1"/>',
half: 13,
      icon: '<path d="M2 14 H19 M25 14 H42"/><path d="M19 7 V21 M25 7 V21" style="stroke-width:2.6"/>' +
            '<path d="M13 14 V3 H17 M27 3 H31 V14"/><rect x="17" y="1" width="10" height="4"/>',
    },
```

`params` follow the nodes (`kw: true`: `key=value`; `optional: true`: left out when blank); `symbol` is drawn between (0, 0) and (80, 0), `icon` is 44 × 24; `conducts: false` marks a current source.

1. **Check it** against theory and a built-in equivalent, with every chatter method (`tests/test_emtsim.cpp`, and a section like 10.5).

10.3 The element interface

Only the first group is required; the defaults (no contribution) apply otherwise.

when	C++ (`Element`)	what it does
always	`stamp(g)`	its k × k conductance
always	`history(t, h)`	its k history currents for the step ending at t
always	`update(t, v)`	keep the solution of the step
always	`currents(out)`, `currentNames()`	recorded columns (default: one, its name)

when	C++ (`Element`)	what it does
it has memory	`saveState` , `loadState`	state for interpolation and chatter removal, in the same order
it has memory	reads `config::euler` , `config::step`	backward-Euler half steps, step length
it fixes a voltage	`knownTerminals` , `knownVoltages` , `setKnownCurrents`	ideal sources
it is non-linear	`nonlinear()` , `check(t, v)`	change segment and re-solve the step
it switches	`hasEvents` , `timeEvent` , `beginStep` , `eventAlpha` , `applyEvent`	switching instants within a step
steady-state start	`phasor` , `initPhasor` , `frequencies`	its admittance and sources at w, its state from the phasors
it adapts	`settlePhasor` , `adjust`	loads and machines that follow the solution

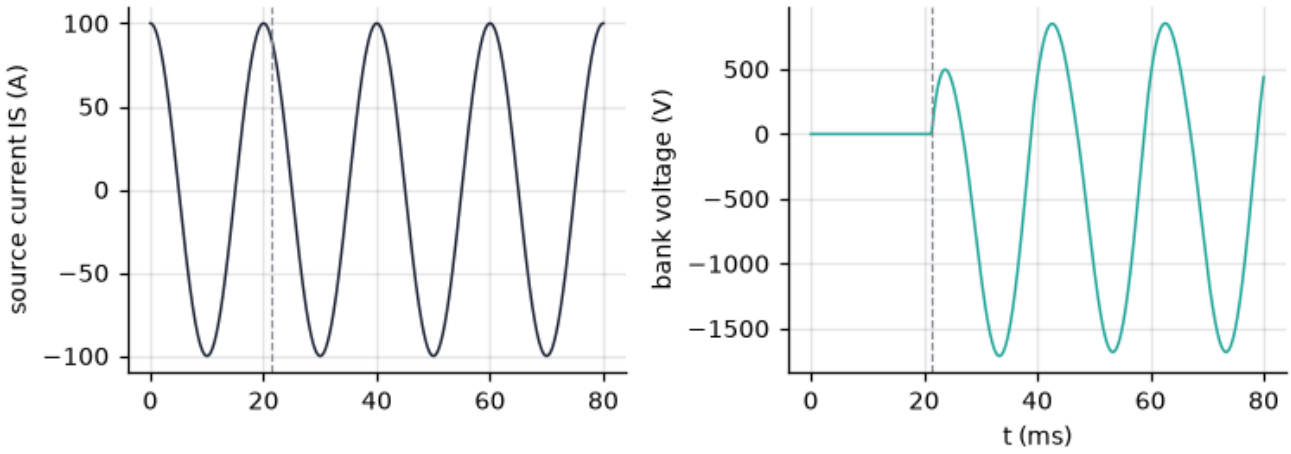
Details: the header of `src/elements.h` ; worked examples: `src/elements.cpp` .

10.4 What the reader checks

- **Keyword:** upper-case letters, digits, `_` , starting with a letter; not a built-in element or block or another component's (a clash stops every run, naming the file).
- **Nodes:** at least the registered number; then values and `key=value` pairs.
- **Values:** the component checks them; its exceptions are reported with the netlist line.

10.5 Checks of the example components

Tests: `I` (2 A, 30°, 50 Hz into 10 Ω: v = R·i at every step); `CR` in `circuits/examples/test_components.net` with every chatter method against the built-in C ‖ R (round-off expected through switching, interpolation and half steps); `CR` at 500 Hz in steady state; `circuits/examples/test_components_steady.net` with `init=steady` at 50 and 250 Hz (everything must repeat after 20 ms). **Theory:** `CR` 's admittance is 1/R + jω℃; against 1/R + jωC the voltage differs by $\omega CR/\sqrt{1 + (\omega CR)^2)\cdot(\omega\Delta t)^2/12} = 0.00196$.



case	reference	error (pu)
I into 10 Ω	v = R i, i = 2 cos(ωt + 30°)	3.52×10^{-15}
CR vs C ‖ R, test_components.net	the built-in pair, every chatter method	1.27×10^{-14}
CR, 1 A at 500 Hz, Δt = 50 µs	phasor 1/(1/R + jω℃)	1.83×10^{-14}
	continuous phasor 1/(1/R + jωC): warping	0.00196
test_components_steady.net	periodic from the first step, v(t + 20 ms) − v(t)	2.88×10^{-15}

- `I` **and** `CR` reproduce their models to round-off; `CR` equals the built-in C ‖ R with every chatter method; the steady start holds both frequencies. The only deviation from the continuous circuit is the warping.

11. Switching studies from the textbooks

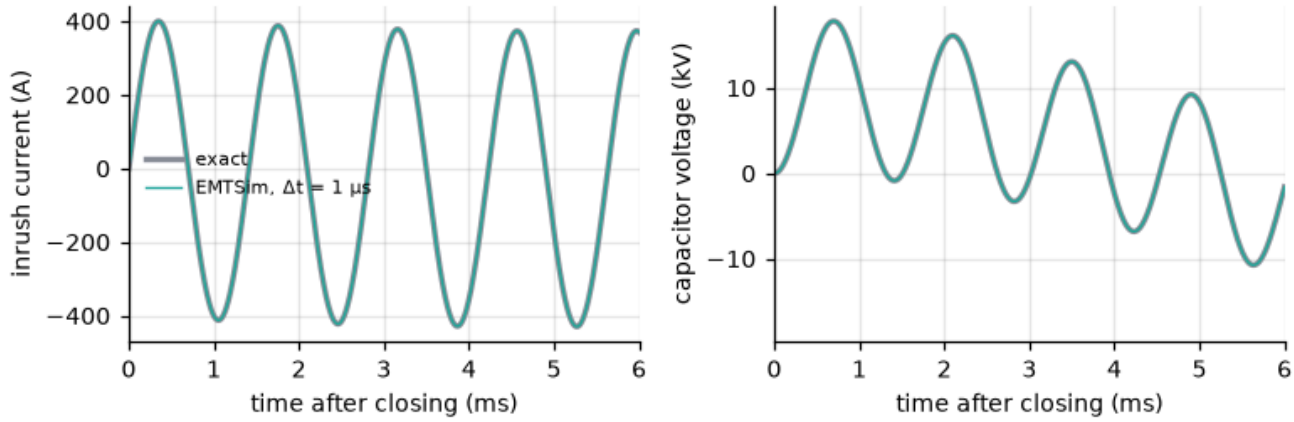
Four classic studies, each against its textbook formula and against the exact solution of the same circuit, solved in closed form between switchings (`harness.exact_linear`).

11.1 Capacitor bank energising

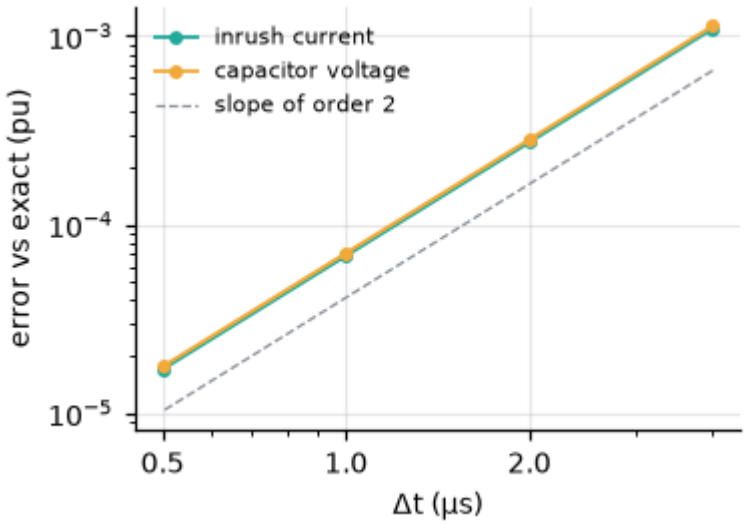
Test: an 11 kV bank (Vm = 8981 V) of 10 µF switched at a voltage peak onto Ls = 5 mH:

```
.tran 1e-6 0.03
V 0 1 8981 0 50       # E     11 kV phase peak, ideal
L 1 2 0.005         # LS
SW 2 3 open 0.02 ron=0.01 roff=1e12   # BRK closes at a voltage peak
C 3 0 1e-05         # C1
```

Theory: the current rings at $f_0 = 1/(2\pi\sqrt{(LsC)}) = 711.8$ Hz with peak Vm/Z_0 ($Z_0 = \sqrt{(Ls/C)} = 22.36\ \Omega$), plus the bank's 50 Hz charging current $Vm\omega C/(1 - \omega^2 LsC)$; the capacitor overshoots to about 2Vm. The exact reference solves the R–L–C loop in closed form from the closing instant (`harness.exact_linear`).



quantity	textbook	exact solution	EMTSim, Δt = 1 µs
inrush peak (A)	$Vm/Z0 = 401.6$; with the 50 Hz charging current $Vm/Z0 + Vm\omega C/(1-\omega^2 LC) = 430.0$	430	430
frequency (Hz)	$1/(2\pi\sqrt{(LsC)}) = 711.8$	–	712
capacitor peak (kV)	$\approx 2Vm = 17.96$	17.8	17.8

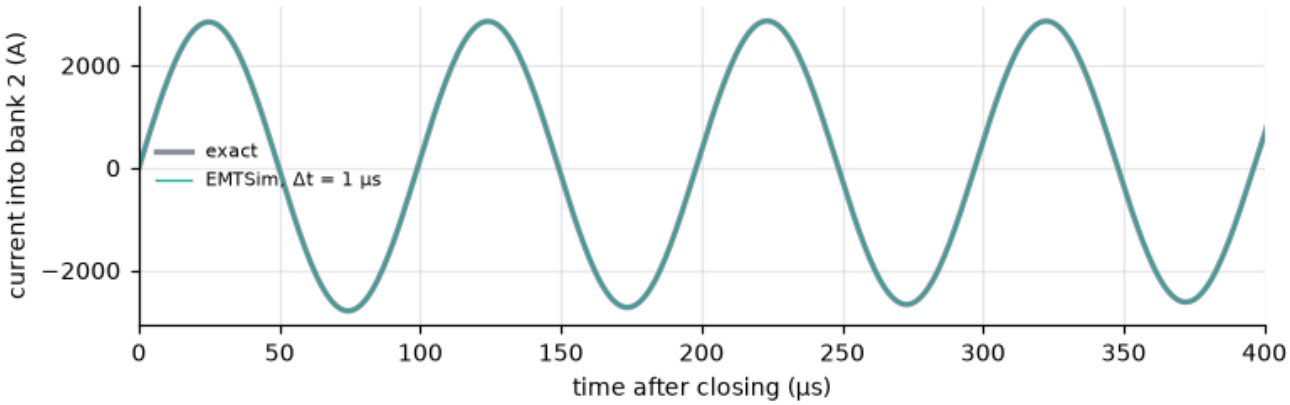


- **Textbook:** Vm/Z_0 alone is 7 % low (it leaves out the charging current); with it, within 0.05 %.
- **Waveform:** order 2, 1.71×10^{-5} of the peak at 0.5 μs: the warping $(\omega_0\Delta t)^2/12$.

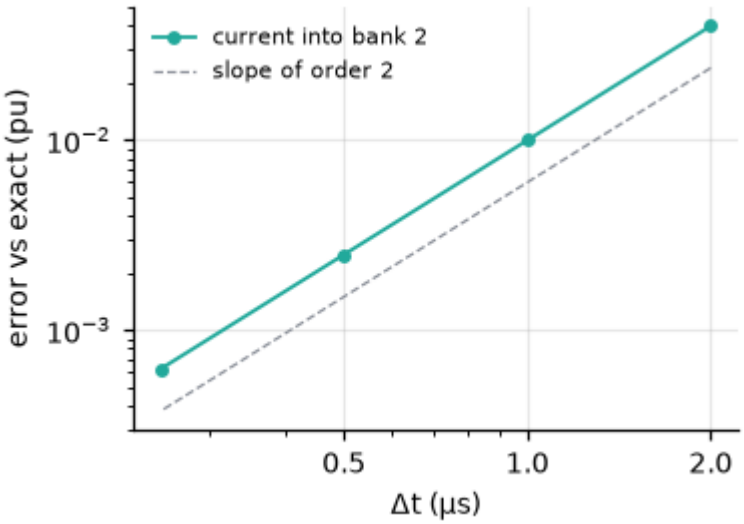
Back-to-back: a second 10 μF bank closed onto the first through Lb = 50 μH at a voltage peak (40 ms; 2 Ω of source resistance has damped the first bank's energising):

```
.tran 1e-6 0.046
V 0 1 8981 0 50       # E
R 1 2 2           # RS    damps the first bank's own energising
L 2 3 0.005         # LS
C 3 0 1e-05        # C1    bank 1, on from the start
SW 3 4 open 0.04 ron=0.01 roff=1e12   # BRK  bank 2 closes at a voltage peak
L 4 5 5e-05        # LB    between the banks
C 5 0 1e-05        # C2
```

Theory: bank 1 discharges into bank 2: peak $\Delta V/\sqrt{(Lb/Ceq)} = 9028/3.16 = 2855$ A at $f_b = 1/(2\pi\sqrt{(LbCeq)}) = 10066$ Hz, $Ceq = C_1C_2/(C_1 + C_2)$; exact reference: the four-state circuit.



quantity	textbook	EMTSim, Δt = 1 μs
inrush peak (A)	2855	2866
frequency (Hz)	10066	10056



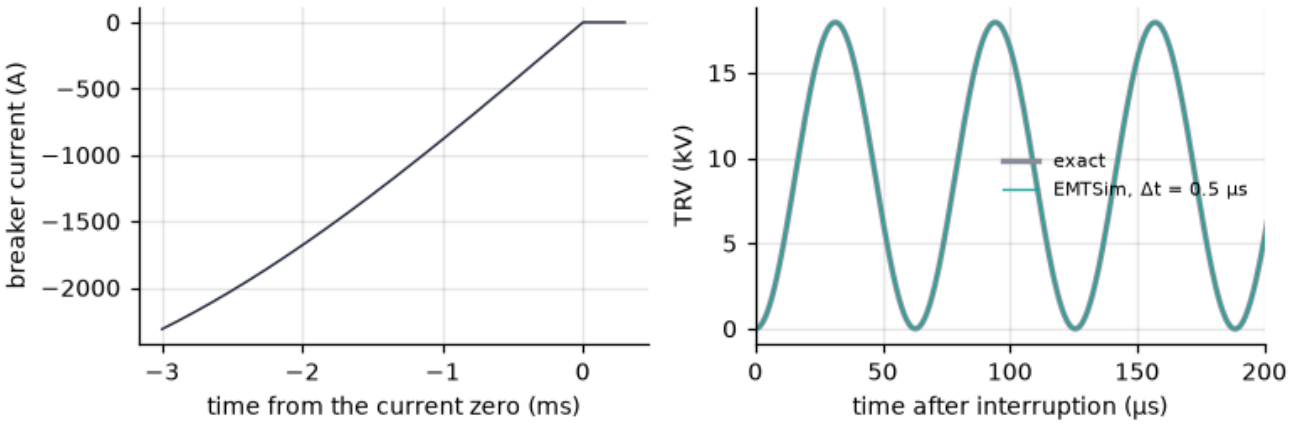
- **6.7× the single bank's inrush at 14× its frequency;** the textbook formula is within 0.4 %.
- **At 10 kHz Δt matters** ($\omega_0\Delta t$ = 0.063 at 1 μs): order 2, measured over the first five cycles.

11.2 Transient recovery voltage of a terminal fault

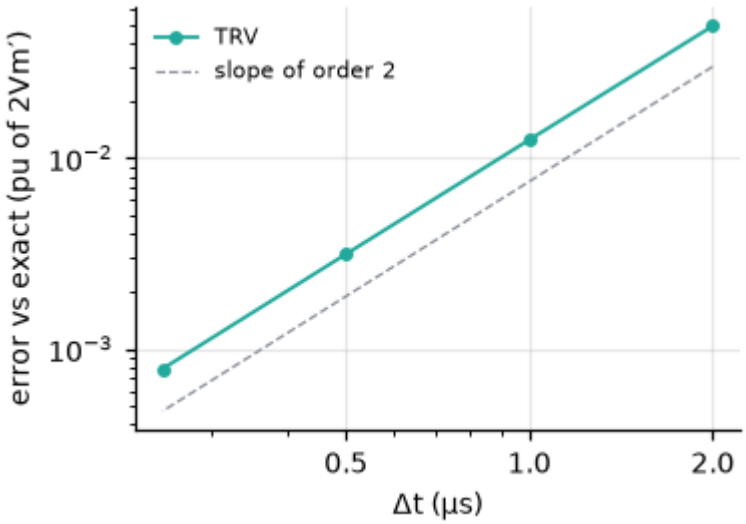
Test: a breaker clears a terminal fault behind Ls = 10 mH with stray Cs = 10 nF, the open command at 15.1 ms waiting for a current zero; steady start (`init=steady`), and the open breaker ideal (`roff=1e12` ; the default 1 MΩ would damp the TRV about 3 % per 300 μs):

```
.tran 0.5e-6 0.0204
.options init=steady
V 0 1 8981 0 50       # E
L 1 2 0.01         # LS    source inductance
C 2 0 1e-08          # CS    stray capacitance at the breaker
SW 2 0 closed 0.0151 czero ron=0.001 roff=1e12   # BRK  a terminal fault, cleared at a current zero
```

Theory: the inductive current's zero falls at a voltage peak; the TRV is $v \approx Vm'(1 - \cos \omega_0 t)$, $\omega_0 = 1/\sqrt{(LsCs)}$ (15.9 kHz), $Vm' = Vm/(1 - \omega^2 LsCs)$: peak $2Vm' = 17962$ V at $\pi/\omega_0 = 31.4$ μs, average rise 572 V/μs. The exact reference finds the current zero (19.9990 ms) and then solves the L–C circuit.



quantity	textbook	EMTSim, Δt = 0.5 μs
TRV peak (V)	17962	17955
time to peak (μs)	31.42	31.51
average rate of rise (V/μs)	572	570



- **Interruption at the current zero** (interpolated), no chatter; CDA's half steps start the TRV about 5 V (of 18 kV) off at 0.5 μs, second order.
- **Peak and timing** as the textbook (undamped); **waveform** order 2: TRV studies need $\Delta t \lesssim 1$ μs.

11.3 Line energising: the Bewley lattice

Test 1: a lossless line (Zc = 300 Ω, τ = 100 μs = 10 steps), open at the far end, from 1 V DC behind Rs = Zc/3:

```
.tran 1e-05 0.0012
V 0 1 1 0 0 100     # E     DC source behind Rs = Zc/3
TL 1 2 300 0.0001        # LINE  lossless, open at node 2
```

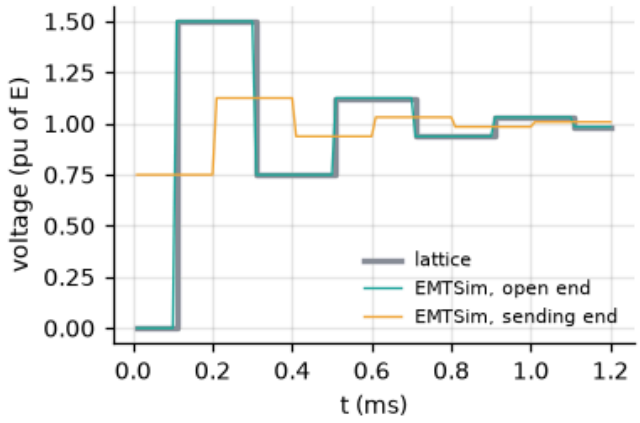
Theory: v1 = E·Zc/(Zc + Rs) = 0.75 V enters; Γr = 1, Γs = -0.50; the open end is the staircase $2v1 \cdot \Sigma(\Gamma s \Gamma r)^m$, a new level every 2τ:

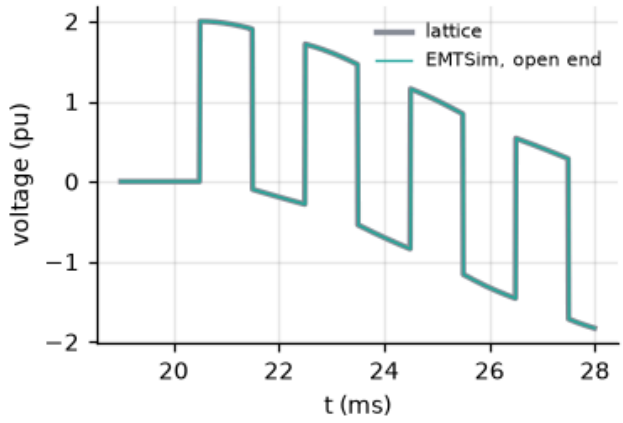
after (μs)	lattice (V)	EMTSim (V)
200	1.5000	1.5000
400	0.7500	0.7500
600	1.1250	1.1250
800	0.9375	0.9375
1000	1.0312	1.0312

Test 2: a 1 pu, 50 Hz source closed at its peak onto a 150 km line (τ = 0.5 ms):

```
.tran 1e-05 0.03
V 0 1 1 0 50       # E     ideal
SW 1 2 open 0.02 ron=0.01 roff=1e12   # BRK closes at the voltage peak
TL 2 3 300 0.0005        # LINE  150 km, open at node 3
```

The same lattice with the sinusoidal source and Γs ≈ −1 gives the open-end voltage; first peak 2.000 pu.





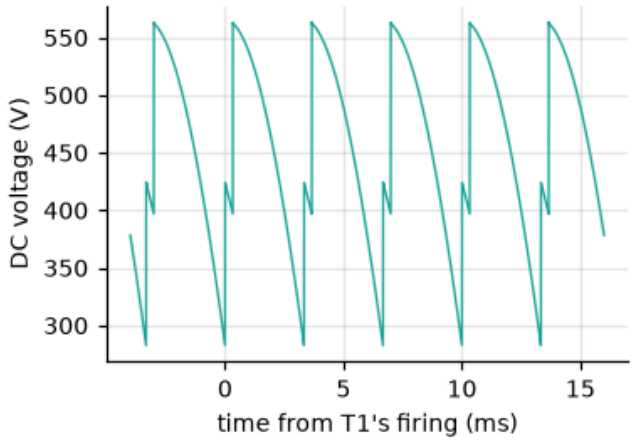
- **Both equal the lattice to round-off** away from the jumps (8.88×10^{-16} DC, 2.19×10^{-9} pu at 50 Hz): with τ a whole number of steps the Bergeron model is exact.
- **The 2 pu switching overvoltage** appears as the textbook says (arresters, closing resistors or closing near a voltage zero reduce it).

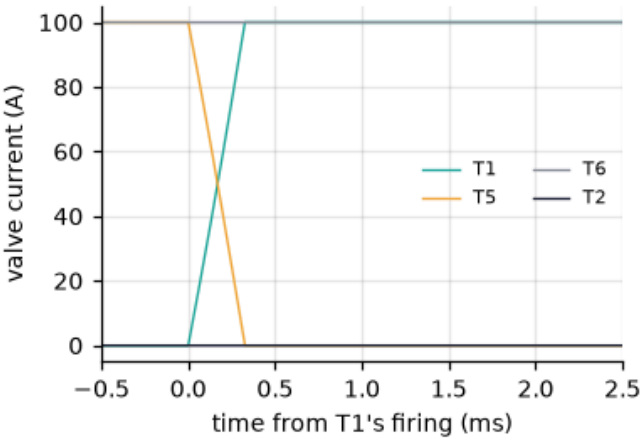
11.4 Six-pulse thyristor bridge with commutation overlap

Test: 400 V, Lc = 0.5 mH per phase, a six-pulse thyristor bridge carrying a constant 100 A, α = 0°, 30°, 60° (gate references as in `circuits/examples/rectifier.net`); for α = 30°:

```
.tran 2e-06 0.12
V 0 1 326.6 0 50 0.0001 0.0005      # SA
V 0 2 326.6 -120 50 0.0001 0.0005   # SB
V 0 3 326.6 120 50 0.0001 0.0005    # SC
THY 1 5  30.0 50 ref=60 width=120 ron=0.001 roff=1e6     # T1
THY 4 3  30.0 50 ref=0 width=120 ron=0.001 roff=1e6      # T2
THY 2 5  30.0 50 ref=-60 width=120 ron=0.001 roff=1e6    # T3
THY 4 1  30.0 50 ref=-120 width=120 ron=0.001 roff=1e6   # T4
THY 3 5  30.0 50 ref=-180 width=120 ron=0.001 roff=1e6   # T5
THY 4 2  30.0 50 ref=120 width=120 ron=0.001 roff=1e6    # T6
I 5 4 100 0 0          # ID    constant DC current
R 5 4 10000            # RSH   keeps the DC side defined
```

Theory: during the overlap μ two valves conduct and the DC voltage loses a notch: $V_d = (3\sqrt{2}/\pi)V_{LL}\cos\alpha - (3/\pi)\omega L_c I_d - 2(r_{on} + R_c)I_d$, and $\cos\alpha - \cos(\alpha + \mu) = 2\omega L_c I_d/(\sqrt{2}\cdot V_{LL})$.





α (°)	V_d textbook (V)	V_d EMTSim (V)	difference (pu)	μ textbook (°)	μ EMTSim (°)
0	524.96	524.99	4.97×10^{-5}	19.19	19.18
30	452.59	452.57	-4.45×10^{-5}	5.86	5.87

α (°)	V_d textbook (V)	V_d EMTSim (V)	difference (pu)	μ textbook (°)	μ EMTSim (°)
60	254.87	254.87	-2.59×10^{-5}	3.61	3.62

- **V_d and μ match the textbook at every α**; the overlap shrinks as α grows. The remainder is the formula's constant resistive drop.

Summary of all checks

81 comparisons with theory; all pass.

- **error:** the measured deviation in the unit of that check (pu of the stated reference, a count, or an absolute difference from the expected order).
- **limit:** the bound it must stay within, set from the theory of the deviation's cause.

component	quantity	theory	simulated	error	limit	result	cause of the deviation
L (trapezoidal)	RL step, Δt = τ/80, vs the trapezoidal recursion	recursion	sim	8.35×10^{-15}	1.00×10^{-12}	pass	round-off
L (trapezoidal)	RL step, order of convergence vs response delayed Δt/2	2	1.99	0.0149	0.05	pass	trapezoidal rule, second order
L (trapezoidal)	RL step, order of convergence vs undelayed step	1 (start-up)	0.99	0.00904	0.05	pass	zero history at t = 0: the step is seen Δt/2 late
L, C (trapezoidal)	LC frequency vs (2/Δt) atan(ω0Δt/2)	1342.35 Hz	1342.35 Hz	9.00×10^{-15}	1.00×10^{-9}	pass	round-off
L, C (trapezoidal)	LC energy, 40 cycles	constant	drift 1.34×10^{-13}	1.34×10^{-13}	1.00×10^{-9}	pass	round-off: the rule conserves the energy of a lossless circuit
L, C (trapezoidal)	AC steady state vs phasor at ω_trap = (2/Δt)tan(ωΔt/2)	phasor	sim	1.92×10^{-13}	1.00×10^{-9}	pass	round-off (and the decayed transient)
V	node voltage = E(t) (ideal) or divider (Norton)	E cos(ωt + φ)	sim	2.13×10^{-16}	1.00×10^{-12}	pass	round-off
V3 + sequence impedance	fault currents vs phasor solution at ω_trap	phasor	sim	2.36×10^{-12}	1.00×10^{-9}	pass	round-off
SW	closing, plain trapezoidal: simulation vs the algorithm applied by hand	reference	sim	3.59×10^{-16}	1.00×10^{-9}	pass	round-off
SW	closing, plain trapezoidal: effective delay vs mechanism estimate	85.06 µs	85.06 µs	9.97×10^{-6}	0.01	pass	plain loop: new state for the whole step, then Δt/2
SW	closing, interpolation only: simulation vs the algorithm applied by hand	reference	sim	2.24×10^{-16}	1.00×10^{-9}	pass	round-off
SW	closing, interpolation only: effective delay vs mechanism estimate	106.88 µs	106.98 µs	5.29×10^{-4}	0.01	pass	trapezoidal start-up Δt/2
SW	closing, chatter removal (CDA) only: simulation vs the algorithm applied by hand	reference	sim	1.01×10^{-11}	1.00×10^{-9}	pass	round-off
SW	closing, chatter removal (CDA) only: effective delay vs mechanism estimate	189.47 µs	190.12 µs	0.00324	0.01	pass	next time point + backward-Euler half steps
SW	closing, both (default): simulation vs the algorithm applied by hand	reference	sim	1.01×10^{-11}	1.00×10^{-9}	pass	round-off
SW	closing, both (default): effective delay vs mechanism estimate	11.29 µs	12.04 µs	0.00378	0.01	pass	backward-Euler half steps
SW czero	opening at a current zero, interp + CDA: inductor voltage afterwards	≈ 3.14×10^{-4} V (leakage)	3.25×10^{-4} V	3.25×10^{-6}	1.00×10^{-4}	pass	R_off = 1 MΩ leakage

component	quantity	theory	simulated	error	limit	result	cause of the deviation
F	fault current with DC offset while applied, default options	exact	sim	2.91×10^{-5}	1.00×10^{-4}	pass	trapezoidal Δt^2 and the CDA half steps at the fault instant
F	1 ms after clearing (τ = 0.1 ms = 2Δt), default options	exact	sim	0.0105	0.02	pass	transient of only two time steps: backward-Euler and trapezoidal error at x = $\Delta t/\tau$ = 0.5
PI	open-end voltage (Ferranti) vs nominal π at ω_trap	1.056283 pu	1.056283 pu	1.74×10^{-9}	1.00×10^{-7}	pass	undecayed 216 Hz line transient, e^(−t/τ) = 1.5×10^{-8} at 1.2 s
RL3, PI3	positive- and zero-sequence impedances vs phasor at ω_trap	phasor	sim	3.16×10^{-12}	1.00×10^{-6}	pass	the DC offset not yet decayed (RL3: e^(−t R/L) after 0.6 s)
TL	lossless line, step: sampled voltages vs the resampled ramp reference	piecewise linear	sim	2.22×10^{-16}	1.00×10^{-12}	pass	round-off
TL	lossy line: first plateau at the open end vs the Bergeron lumped-loss value	0.92915	0.92915	3.33×10^{-16}	1.00×10^{-9}	pass	round-off
TL3	modal arrival times and amplitudes (4/3 at τ1, 2 at τ0 on the energised phase)	piecewise linear	sim	4.44×10^{-16}	1.00×10^{-12}	pass	round-off
XF	ratio, magnetising current, short-circuit current, regulation vs phasor at ω_trap	phasor	sim	9.46×10^{-13}	1.00×10^{-7}	pass	round-off and the 1 GΩ / 1 nΩ open and short
XF saturation	inrush, order of convergence vs the continuous model	2	1.99	0.00761	0.1	pass	trapezoidal rule; the knee corners add only O(Δt^2) locally
XF saturation	inrush first peak at Δt = 50 µs	609.2 A	609.2 A	5.72×10^{-5}	0.001	pass	trapezoidal flux integration (Δt^2)
XF3	vector groups: phase shift and line-voltage ratio (7 connections)	0 / ∓30°	sim	4.58×10^{-7}	1.00×10^{-5}	pass	magnetising current through the leakage of the delta/star windings
MOV	current vs the documented piecewise-linear curve	PWL(v)	sim	6.69×10^{-15}	1.00×10^{-9}	pass	round-off
MOV	current vs the power law i = Iref (v/Vref)^q	≤ 0.86 %	max 0.830 %	0	1.00×10^{-6}	pass	linear segments between break points 30 % apart in current
D, THY	RL rectifier: extinction angle and mean voltage, interpolation on	analytical	sim	0.00156	0.01	pass	trapezoidal Δt^2 and the CDA half steps (degrees, or % of Vm)
THY	RL rectifier: extinction angle, interpolation off	analytical	sim	0.221	0.9	pass	firing at the next time point: up to one step = 0.9°
SM	steady-state load angle vs the phasor solution at the trapezoidal reactance	analytical	sim	7.90×10^{-6}	0.001	pass	settling of the damped swing (degrees)
SM	steady-state electrical power = mechanical power	Pm	sim	5.51×10^{-7}	1.00×10^{-4}	pass	settling of the damped swing (pu)
SM	swing frequency vs the eigenvalue of the linearised continuous model	2.2963 Hz	2.2959 Hz	1.38×10^{-4}	5.00×10^{-4}	pass	amplitude of the 2° swing, $-(\Delta\delta)^2/16$
SM	swing growth rate vs the eigenvalue (negative damping from r_a with stator transients)	0.01342 1/s	0.01342 1/s	5.43×10^{-4}	0.01	pass	fit of the peak envelope

component	quantity	theory	simulated	error	limit	result	cause of the deviation
IM	steady-state slip vs the discrete steady state of the model (with the EMF delay)	closed form	sim	8.40×10^{-11}	1.00×10^{-4}	pass	settling
IM	continuous equivalent circuit vs the model's continuous steady state	0.025016	0.025016	3.01×10^{-12}	1.00×10^{-6}	pass	none: the model's steady state is the T-equivalent circuit
IM	slip error vs the equivalent circuit at Δt = 100 μs	0	0.98 %	0.00978	0.02	pass	the one-step delay of the internal EMF e′ (first order in Δt)
VC, IC	output = signal of the same step (solved together)	s(t)	sim	1.75×10^{-15}	1.00×10^{-12}	pass	round-off
VC, IC	coupling=delay: output = signal one step earlier	s(t − Δt)	sim	2.27×10^{-16}	1.00×10^{-12}	pass	round-off
VM, METER	signal = network quantity of the same step	v(t), i(t)	sim	0	1.00×10^{-12}	pass	round-off
VC	phase lag of a 50 Hz controlled source	0°	-1.4e-14°	1.42×10^{-14}	1.00×10^{-9}	pass	none: solved together
VC	coupling=delay: phase lag of a 50 Hz controlled source	ωΔt = 0.900°	0.900°	6.25×10^{-14}	1.00×10^{-6}	pass	the one-step interface delay of TACS and PSCAD
SWC	switch driven by a comparator vs the timed switch (interpolation on)	identical	sim	0	1.00×10^{-12}	pass	round-off: the comparator's crossing instant is exact for a linear input
THYC	thyristor fired by a comparator vs the timed thyristor	identical	sim	1.99×10^{-12}	1.00×10^{-9}	pass	round-off
sources, math, limiters, logic	every sample vs the defining formula (25 blocks)	formula	sim	3.55×10^{-15}	1.00×10^{-12}	pass	round-off
LAG (trap)	step response at Δt/T = 0.1 vs the recursion $y_k = 1 - \rho^k$	3.07×10^{-4}	3.07×10^{-4}	4.44×10^{-16}	1.00×10^{-12}	pass	round-off
LAG (trap)	step response at Δt/T = 1 vs the recursion $y_k = 1 - \rho^k$	0.0345	0.0345	5.55×10^{-17}	1.00×10^{-12}	pass	round-off
LAG (exp)	step response, Δt = T	1 − e^(−t/T)	sim	2.00×10^{-15}	1.00×10^{-12}	pass	round-off: exact for a step
TF (exp)	2nd-order TF with a zero vs scipy lsim (exact for linear-between-samples input)	lsim	sim	6.48×10^{-14}	1.00×10^{-10}	pass	round-off
LP2 (exp)	resonance gain vs G $[\sin(\omega\Delta t/2)/(\omega\Delta t/2)]^2$	1.664475	1.664475	2.34×10^{-10}	1.00×10^{-6}	pass	round-off and the decayed transient
LP2	resonance gain, trapezoidal, vs the bilinear-transform prediction	1.664457	1.664457	2.93×10^{-15}	1.00×10^{-6}	pass	round-off and the decayed transient
TON	on-delay output time	t_prev + T	0.01730 s	3.12×10^{-13}	1.00×10^{-6}	pass	timers count whole steps: the step in which the input turned on counts in full
ZOH	samples held at multiples of ts	u(k ts)	sim	2.03×10^{-14}	1.00×10^{-12}	pass	round-off
RMS, DFT, THD, PQ3, SEQ	steady-state values on exact sinusoids	analytical	sim	1.06×10^{-13}	1.00×10^{-9}	pass	round-off: whole-cycle windows of uniform samples are exact
SOGI	in-phase and quadrature gain and phase vs the bilinear-transform prediction	D(jω_trap)	sim	1.11×10^{-13}	1.00×10^{-7}	pass	round-off and the decayed transient

component	quantity	theory	simulated	error	limit	result	cause of the deviation
PLL	phase error after a 0.5 Hz offset vs the linear 2nd-order loop	Δω/ωd e^(−ζωn t) sin ωd t	sim	2.56×10^{-5}	0.01	pass	sin(e) ≈ e, and the discrete loop at ωnΔt = 0.006
PLL	steady-state frequency	50.5000 Hz	50.500000 Hz	1.18×10^{-12}	1.00×10^{-9}	pass	round-off (type-2 loop)
I	source current and v = R i across a resistor	i = peak cos(ωt + angle)	sim	3.52×10^{-15}	1.00×10^{-12}	pass	round-off
CR	equal to the built-in C ‖ R through a breaker and a diode, 5 chatter methods	C ‖ R	sim	1.27×10^{-14}	1.00×10^{-9}	pass	round-off (one branch instead of two)
CR	500 Hz steady state vs the phasor at the trapezoidal frequency	1/(1/R + jω℄)	sim	1.83×10^{-14}	1.00×10^{-9}	pass	round-off
CR	500 Hz steady state vs the continuous phasor	≈ 0.00196	0.00196	6.01×10^{-4}	0.05	pass	frequency warping of the capacitor, $(\omega\Delta t)^2/12$ of its share
I, CR	steady-state start at 50 and 250 Hz: v(t + T) − v(t)	0	sim	2.88×10^{-15}	1.00×10^{-11}	pass	round-off
study: capacitor bank	inrush peak vs Vm/Z0 + Vm·ωC (textbook)	430.0 A	429.8 A	5.41×10^{-4}	0.005	pass	the formula adds the two peaks; their timing differs a little
study: capacitor bank	inrush frequency vs 1/(2π√(LC))	711.8 Hz	712.5 Hz	9.94×10^{-4}	0.01	pass	the 50 Hz component shifts the zero crossings slightly
study: capacitor bank	waveform vs the exact solution, Δt = 0.5 µs	exact	sim	1.71×10^{-5}	1.00×10^{-4}	pass	trapezoidal warping, order 2
study: back-to-back banks	inrush peak vs ΔV/√(Lb/Ceq) (textbook)	2855 A	2866 A	0.00384	0.03	pass	the formula ignores the source and the losses
study: back-to-back banks	frequency vs 1/(2π√(Lb·Ceq))	10066 Hz	10056 Hz	9.38×10^{-4}	0.01	pass	the source side (Ls ≫ Lb) is nearly an open circuit
study: back-to-back banks	waveform vs the exact solution, Δt = 0.25 µs	exact	sim	6.24×10^{-4}	0.001	pass	warping, order 2
study: TRV	TRV peak vs 2·Vm/(1 − ω²LsCs) (textbook)	17962 V	17955 V	3.72×10^{-4}	0.002	pass	no damping in the circuit: 1 − cos(ω0 t)
study: TRV	time to the TRV peak vs π/ω0	31.42 µs	31.51 µs	0.0031	0.02	pass	sampled every 0.5 µs
study: TRV	waveform vs the exact solution, Δt = 0.25 µs	exact	sim	7.85×10^{-4}	0.001	pass	warping at 16 kHz, order 2
study: line energising	DC staircase at both ends vs the Bewley lattice	lattice	sim	8.88×10^{-16}	1.00×10^{-8}	pass	τ a whole number of steps: exact
study: line energising	50 Hz closing at the peak: open end vs the lattice	lattice	sim	2.19×10^{-9}	1.00×10^{-8}	pass	τ a whole number of steps: exact
study: six-pulse bridge	α = 0°: mean DC voltage vs (3√2/π)V cos α − (3/π)ωLcId	524.96 V	524.99 V	4.97×10^{-5}	0.002	pass	the formula's resistive drop is approximate
study: six-pulse bridge	α = 0°: overlap angle µ	19.19°	19.18°	7.24×10^{-4}	0.03	pass	sampled every 0.036°; ron and Rc neglected in µ
study: six-pulse bridge	α = 30°: mean DC voltage vs (3√2/π)V cos α − (3/π)ωLcId	452.59 V	452.57 V	4.45×10^{-5}	0.002	pass	the formula's resistive drop is approximate
study: six-pulse bridge	α = 30°: overlap angle µ	5.86°	5.87°	0.00125	0.03	pass	sampled every 0.036°; ron and Rc neglected in µ

component	quantity	theory	simulated	error	limit	result	cause of the deviation
study: six-pulse bridge	α = 60°: mean DC voltage vs (3√2/π)V cos α − (3/π)ωLcId	254.87 V	254.87 V	2.59×10^{-5}	0.002	pass	the formula's resistive drop is approximate
study: six-pulse bridge	α = 60°: overlap angle μ	3.61°	3.62°	0.00354	0.03	pass	sampled every 0.036°; ron and Rc neglected in μ